Physics Factsheet

January 2001 Number 12

Applying Newton's Laws

What are Newton's Laws?

Newton's First Law: Every body continues at rest or with constant velocity unless acted upon by a resultant force

Newton's Second Law: The rate of change of momentum of a body is proportional to the resultant force that acts on it

Newton's Third Law: When two bodies interact, the forces they exert on each other are of equal magnitude and opposite direction

1. The First Law

This is mainly used to find unknown forces, by using it in the form:

If a body is at rest or moving with constant velocity, there must be no resultant force on it.

Tip: *It is important to note that the body must be moving with constant* ***velocity****, not just constant* ***speed*** *– the body must move at a steady speed and always in the same direction. A body moving in a circle may move at a steady speed, but still has a resultant force on it.*

A body that is at rest or moving with constant velocity is said to be in **equilibrium**. Factsheets 2 Vectors and Forces and 4 Moments and Equilibrium give details on how to solve problems about bodies in equilibrium, but the general strategy is:

- Draw a diagram, showing **all** the forces on the body.
- Check the body really is in equilibrium– is it stationary or moving with constant velocity?
- Resolve forces in two perpendicular directions – either horizontally and vertically, or if the body is resting on a slope, then parallel and perpendicular to the slope.
- Equate the total force in each direction to zero.
- If necessary, take **moments** – this will be required if not all the forces pass through one point – and equate to zero.
- Solve your equations to find the unknown forces.

We can also use the First Law to help with problems for bodies that are not in equilibrium, provided there is no resultant force on them **in a particular direction**. In cases like this it is used together with the Second Law – see later for examples.

Typical Exam Question

An aircraft of mass 11 000kg, which moves at a constant velocity, v, and constant altitude, is powered by propellers and experiences a drag force.

(a) Draw a labelled free body diagram showing the 4 main forces acting on the aircraft. [4]

(b) The thrust from the propellers is 225kN and the drag force is given by $10v^2$. Calculate the aircraft's level flight speed. [2]

(a) Lift vertically upward ✓ weight vertically downward ✓
thrust forwards ✓ drag backwards ✓

(b) For level flight, horizontal forces are equal:
$225000 = 10v^2$ ✓
$v = 150ms^{-1}$ ✓

Free-Body Force Diagrams

Many questions require you to draw a free-body force diagram; however, even if it is not asked for explicitly, it is **vital** in solving any question involving Newton's Laws. All the diagram shows is the body in which you are interested, together with all the forces acting on it – **not** the forces acting on any other body.

It is a good idea to draw a diagram of the whole situation first, including all the bodies involved, since it helps you to make sure you do not miss out any forces – this is the commonest mistake!

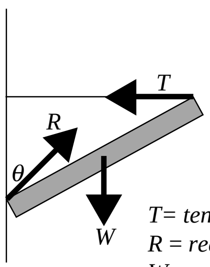
To avoid missing out forces, work through the following check-list:

- The body's weight, acting from its centre of mass.
- If the body is in contact with anything, there will be a normal reaction force. This acts at right-angles to the surface.
- The reaction at a hinge can act at any angle to the surface, so put it in at an unknown angle.
- Friction will act if the body is in contact with a rough surface, and if it is moving or has any "tendency" to move. Having a "tendency" to move means the body would move if there was no friction. Friction acts parallel to the surface, in the direction opposite to the way the body moves or would tend to move.
- If the body is attached to a string, the tension of the string will act on it. Tension always pulls, never pushes.
- If the body is attached to a spring, the tension or thrust of the spring will act on it. It can pull (tension) or push (thrust).
- If the body is moving through the air, then air resistance will act in the opposite direction to the one it is moving in.
- Aircraft experience a lift force vertically upwards.

Example 1. A uniform rod AB is attached at end A to a vertical wall by a hinge. A spring has one end attached to end B of the rod, and the other is fixed to the wall above A so that the spring is horizontal. Draw a free body force diagram to show the forces acting on the rod.

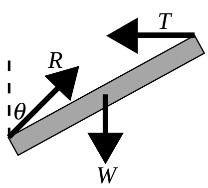
Note: the rod being "uniform" means that its centre of mass is at its centre.

Diagram of the whole thing



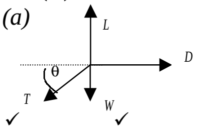
T= tension of spring
R = reaction at hinge
W = weight of rod

Free-body force diagram



Tip*: You must make it clear what any letters you use in your diagrams stand for.*

Typical Exam Question
A speedboat is towing a paraglider at a constant speed and height on the end of a light rope of length 30m, which makes an angle θ with the horizontal. The forces acting on the paraglider are the vertical lift, L, the horizontal drag, D, his weight, W and the tension in the rope, T.
(a) Draw a free body diagram of the paraglider showing the forces L, D, W and T. [2]
(b) State the value of the resultant of these forces. [1]
(c) Hence, write an equation relating the magnitudes of:
(i) D, T and θ [1]
(ii) L, W, T and θ. [1]

(a)


(b) The resultant of these forces must be zero. ✓
(c) (i) Resolving horizontally: $D = T\cos\theta$ ✓
(ii) Resolving vertically: $L = W + T\sin\theta$ ✓ .

Mass and Weight
It is vital to remember the difference between mass and weight. To summarise:

	Mass	Weight
Measures	The amount of matter in a body	The force of gravity on a body
Changes	Not at all, unless the body is broken up	Different depending on the force of gravity – so would be different on the moon, and high above the earth's surface
Unit	kilogram	newton
Scalar/ vector	scalar	vector (since it is a force)

For any body, $W = Mg$, where W is its weight (N), M is its mass (kg) and g is the acceleration due to gravity.

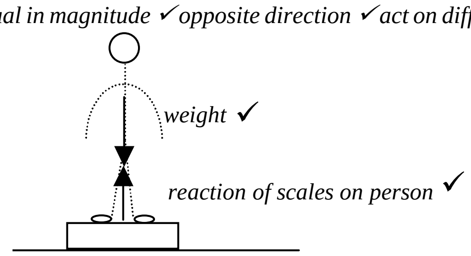
Weight always acts vertically downwards from the centre of mass of the body.

2. The Third Law
We are considering the third law next because a good understanding of both first and third law is necessary to approach some second law problems. Here, we will be applying the third law to bodies in contact, or connected by a rope. It tells us that, for example:
- the downward force you exert on the floor by standing on it is the same size as the upward force the floor exerts on you
- if you walk a dog on a lead, the tension in the lead acting on you (due to the dog tugging you) is of the same magnitude as the tension in the lead acting on the dog, tugging it towards you.
- when you push on a door, the door pushes back on you with the same magnitude force.

Example 2. Two children are playing "tug-of-war". One of them suddenly lets go of the rope. Explain why the other child may fall over, and explain the direction in which s/he falls.

When both children are tugging the rope, the force each child exerts on the rope is equal and opposite the force the rope exerts on the child – the rope pulls each child forward, and the child tries to pull the rope backwards. When one child lets go, the tension in the rope is removed. The other child is still exerting the same backward pull on the rope, but there is now no compensating forward pull on the child. So the child falls backwards.

Typical Exam Question
(a) Identify three properties of pairs of forces that are linked by Newton's third law. [3]
(b) A person stands on bathroom scales on the ground. Draw a free-body force diagram for the person. Identify all forces clearly. [2]
(c) For the situation in (b), state the other force forming a Newton's third law pair with the reaction force of the scales acting on the person's feet. [1]

(a) equal in magnitude ✓ *opposite direction* ✓ *act on different bodies* ✓
(b)


(c) The person's weight ✓

3. The Second Law
Newton's Second Law is most often used in the form

$F = ma$
F = resultant force (newtons) m = mass (kilograms)
a = acceleration (metres seconds^{-2})

See Factsheet 9 Momentum for more on using the second law in its other form.

This form is **only valid if mass is constant**. However, since the examination does not require you to consider variable mass, this is not a problem.

Note that both F and a are **vectors** – the force determines not only the **size** of the acceleration, but also its **direction** – a body accelerates in the direction of the resultant force on it.

Second law problems – like any other mechanics problem – require you to draw a clear diagram, including all the forces acting on a body. You then need to:
- Resolve forces in the direction in which acceleration is taking place, and use F = ma, where F is the **resultant force** in that direction.
- If necessary, resolve forces **perpendicular** to the direction of acceleration, and use the fact that the resultant force is zero.
- If two bodies are involved, use the third law to identify equal forces

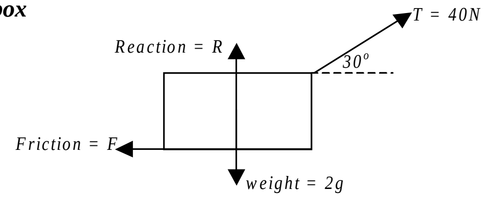
The following examples illustrate common ways of using the second law.

Showing the second law is equivalent to F = ma
The second law states that force is proportional to the rate of change of momentum.
Since momentum is given by mass × velocity, this means force is proportional to the rate of change of (mass × velocity).
If we assume mass is constant, then this becomes:
mass × (rate of change of velocity)
But rate of change of velocity is acceleration.

So we have force is proportional to mass × acceleration – or, as an equation, $F \propto ma$, or $F = kma$, where k is a constant of proportionality.

We get rid of the constant k by **defining** the **newton** to be such that a force of 1N gives a mass of 1kg an acceleration of 1ms^{-2}

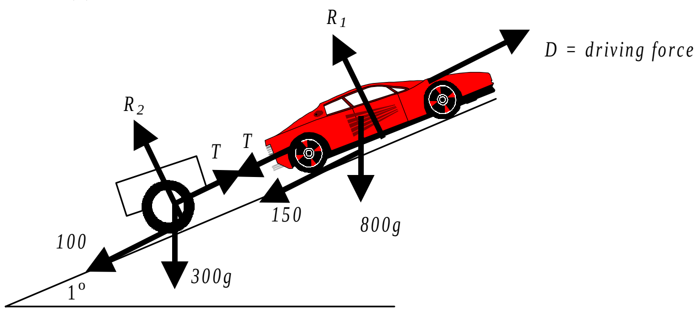
Example 3. A box of mass 2kg is being towed along a rough horizontal surface by a person pulling it on a string. The string is at 30^o to the horizontal, and its tension is 10N. The box is accelerating at $1.0ms^{-2}$. Taking $g = 9.81ms^{-2}$, find:
(a) the frictional force acting on the box.
(b) the magnitude of the normal reaction force exerted by the ground on the box



(a) We need to resolve in the direction of the acceleration – which is horizontal, since the box is moving on horizontal ground:
$T\cos30^o - F = ma = 2 \times 1$
So $F = 10\cos30^o - 2 = 6.66N$ (3 SF)

(b) To find any other information, we need to resolve perpendicular to the direction of acceleration, and use the fact that resultant force is zero:
$T\sin30^o + R - 2g = 0$
So $R = 2g - 10\sin30^o = 9.62N$ (3SF)

Example 4
A car of mass 800kg is towing a trailer of mass 300kg up a road inclined at 1^o to the horizontal. The car exerts a constant driving force, and starting from rest, achieves a speed of $10ms^{-1}$ in 50 seconds. The frictional forces on the car and trailer are constant, and of magnitude 150N and 100N respectively. Take $g = 10ms^{-2}$
Find: (a) the driving force of the car
(b) the tension in the tow bar.



(a) We first need to work out the acceleration from the information given. Since all the forces are constant, we know the acceleration will be constant, so constant acceleration equations can be used:
Using $v = u + at$: $10 = a \times 50 \Rightarrow a = 0.2ms^{-2}$

We now need to use F = ma. We must resolve in the direction of the acceleration – that is, up the hill

We can consider the car and trailer together – this will avoid bringing in the tension in the tow rope:
$D - 800g\sin1^o - 300g\sin1^o - 100 - 150 = (800 + 300)a = 1100 \times 0.2$
So $D = 1100 \times 0.2 + 800g\sin1^o + 300g\sin1^o + 100 + 150 = 662N$

> ***Tip:*** *You could consider the car and trailer seperately – you would then get two equations, which you would have to solve to find D*

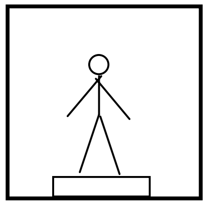
.(b) Since we need the tension here, we must look at either the car or the trailer – it doesn't matter which.
Trailer: $T - 100 - 300g\sin1^o = 300 \times 0.2 \Rightarrow T = 212N$

> ***Tip: Many students lose marks by ignoring the weight of the car.*** *If a hill is involved, the weight will come in to your equations!*

Example 5. A slimming club is situated at the top of a tall building; to motivate its clientele, the club has installed its own lift which contains a weighing machine. The lift accelerates uniformly at $1.0ms^{-2}$ for 90% of its journey, both going up and coming down.
Taking $g = 10ms^{-2}$, calculate:
(a) The resultant force required to accelerate a person whose mass is 80kg at $1.0ms^{-2}$.
(b) The reading (in kg) on the weighing machine when the 80kg person stands on it as the lift accelerates upwards.
(c) The reading (in kg) of the machine when the same person stands on it as it accelerates downwards.

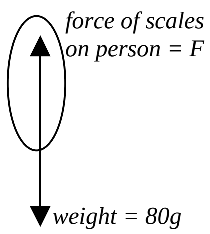
(a) We use F = ma: $F = 80 \times 1 = 80N$

(b) To work out the reading on the scales, we need to consider the forces acting on the ***person***



They are in contact with the scales, which are pushing them upwards. Their weight acts downwards.

So for the person, we have



We also know that the resultant force on the person is 80N upwards, from (a). This gives us:
$F - 80g = 80$

Now the reading on the scales is worked out from the force the person exerts on the scales.

The third law tells us that the force exerted by the person on the scales is the same in magnitude as the force exerted by the scales on the person – so it is F = 80g + 80 downwards.

The scales are calibrated to give a mass reading, rather than a "weight" reading.

So the mass reading on the scales will be obtained by dividing the force reading by g.

So the mass reading is $F \div g = (80g + 80) \div g = 88kg$

(c) When the lift is moving downwards, the resultant force is 80N downwards, so we have: $80g - F = 80$.

This gives $F = 80g - 80$, *and hence a mass reading of* $(80g - 80) \div g = 72kg$

Experimental investigation of Newton's Second Law

To investigate Newton's Second Law, a trolley on a track with ticker-tape is used.

First, the track is friction-compensated by tilting it until the trolley will run down at a steady speed – this occurs when the dots on the ticker-tape are evenly spaced.

A piece of elastic is attached to the trolley. A person pulls on the elastic so that it is always kept at the same length as the trolley moves - this provides a constant force on the trolley.

Different forces can be investigated by using two or three identical pieces of elastic. Different masses can be investigated by stacking trollies on top of each other.

To analyse the results, the ticker-tape is cut into 10 – dot lengths. Since 50 dots are produced every second, this allows the average velocity every 0.2 seconds to be calculated. From these velocities, the acceleration can be calculated.

The variation of the acceleration with applied force (number of pieces of elastic) and with mass (number of trollies) can then be analysed.

Questions

1. State Newton's second law, and explain how it leads to "F = ma"

2. Define the newton

3. Explain why the first law refers to "constant velocity", not "constant speed"

4. State three characteristics of a Newton's third law pair of forces.

5. Explain why, if you are leaning on a shut door and it suddenly opens, you fall over.

6. A car of mass 700kg is moving at a steady speed up a slope inclined at 2^o to the horizontal. The frictional resistance to motion is constant, and of magnitude 100N. Take $g = 10ms^{-2}$
   a) Calculate the driving force of the car

   The car now travels down the same slope, with the engine exerting the same driving force.
   b) Calculate the acceleration of the car.

7. A person of mass 60kg is standing in a lift of mass 250kg. The lift accelerates upwards at $1.5ms^{-2}$. Taking $g = 9.8ms^{-2}$, calculate
   a) the tension in the lift cable
   b) the force exerted on the person by the floor of the lift

Answers

1. –4. answers may be found in the text
5. The door was pushing back on you with a force equal in magnitude to the force you exerted on it. When the door is opened, this force is removed, so the forces on you are unbalanced, so you fall towards the door.
6. a) 344N b) $0.698ms^{-2}$ (both 3SF)
7. a) 3500 N (3SF) b) 678N

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Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

A manned rocket consists of a main rocket with spacecraft attached, with combined mass of 1.0×10^4kg, and a separable propulsion unit complete with its own first stage motor and fuel. The mass of this separate unit is 500 kg.
Both parts of the rocket contain motors capable of producing a constant thrust of 1.2×10^5N and are used in turn, the one in the main body igniting as soon as the separate unit has run out of fuel and been jettisoned. The rocket takes off from the ground and continues to fly vertically upwards. Take $g = 9.8ms^{-2}$

(a) Ignoring the effects of air resistance, calculate the:
(i) resultant force on the rocket at the instant of take-off. [2]
$1.2 \times 10^5 – (1\times 10^4 + 500)g$ ✓ = 15000N ✗ 1/2

The student knows the correct calculation to carry out and has used the correct method, but has lost the final mark by using $g = 10ms^{-2}$ instead of $9.8ms^{-2}$ as it says. Read the question!

(ii) initial acceleration of the rocket. [2]
$F = ma$
$15000 = (1 \times 10^4 + 500)a$ ✓ so $a = 1.4286ms^{-2}$ ✗ 1/2

Again, no difficulties understanding what is required, and the student could have gained full marks on this section from using the wrong answer to a) i) correctly, but the final mark is lost through using an inappropriate number of significant figures.

(b) Rocket fuel is burned at a steady rate of 2.5kg s^{-1}. The first stage motor has 200kg of fuel available. Calculate the:
(i) time taken to use up the fuel in the first stage. [1]
$200 \div 2.5$ = 80 seconds ✓ 1/1
(ii) acceleration of the rocket at the instant just before the first stage fuel runs out.
$15000 = (1 \times 10^4 + 500 – 200)a$, ✓ so $a = 1.46ms^{-2}$ ✓ 2/2

The student has gained full marks here, despite having a numerically incorrect answer, since the answer to a) i) has been used correctly and the answer is given to a suitable number of significant figures.

(c) The rocket motor in the main body ignites and begins to supply full thrust of 1.2×10^5N at the instant the first stage finishes and falls away. For the instant in time just after separation of the stages, calculate the:
(i) acceleration of the main rocket. [2]
$1.2 \times 10^5 - 1 \times 10^4 g = 1 \times 10^4 a$. ✓
$a = 11ms^{-2}$ ✗ 1/2

Again, the student understood the calculation to be carried out, but has made a numerical error. Since s/he showed working, a mark can be awarded. The size of the answer – so different from earlier figures – should have alerted the student to the fact that the answer was wrong.

(ii) resultant force on an astronaut of mass 70kg aboard the spacecraft. [2]
$mg – ma = 80g – 80 \times 11 = -80N$???? 0/2

Had the student drawn a diagram and worked out the resultant force carefully, s/he probably would not have written "mg – ma" instead of "mg + ma". The student clearly realises the negative answer is wrong, but should therefore have checked previous work.

Examiner's answers
(a) (i) $1.2 \times 10^5 - (1 \times 10^4 + 500) \times 9.8 = 17100$ N
(ii) $17100 = 10500\,a$ ✓ $1.69\ ms^{-2} = a$ ✓
(b) (i) 200/2.5 = 80 seconds ✓
(ii) $17100 = 10300\,a$ ✓ $a=1.7\ ms^{-2}$ ✓
(c) (i) $1.2 \times 10^5 - 10^4 \times 9.8 = 10^4 \times a$ ✓ $2.2\ ms^{-2} = a$ ✓
(ii) $F = mg + ma$ ✓
$= 70 \times 9.8 + 70 \times 2.2 = 840$ N ✓