Physics Factsheet

April 2001 Number 17

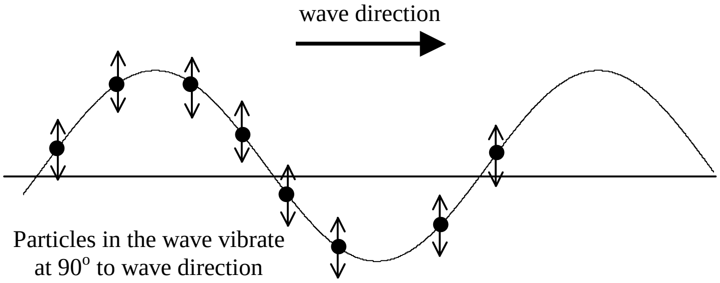
Waves Basics

This Factsheet will introduce wave definitions and basic properties.

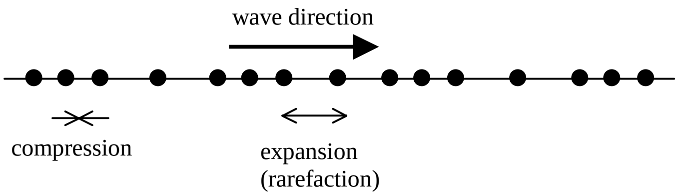
Types of wave

Waves may be **mechanical** (i.e. they require a medium such as air or water to propagate) or **electromagnetic** (which propagate in a vacuum) Waves can be classified as:

a) **Transverse** – here the disturbance is at right angles to the direction of the wave. Examples include water waves and light (electromagnetic).



b) **Longitudinal** – here the disturbance is in the same direction, parallel to the direction of the waves. Examples include sound and seismic P waves.



Particles of the wave vibrate in the same direction as the wave

Both longitudinal and transverse waves can be represented graphically as shown below.

Exam Hint: *You will be expected to know examples of longitudinal and transverse waves and to describe the differences between them.*

Wave pulses and continuous waves

- A wave pulse involves a short or single disturbance of the medium it is travelling in. For example, dropping an object in water may produce a wave pulse.
- Continuous waves involve repeated disturbances of the medium. For example, to produce continuous waves in a ripple tank, the dipper would have to be dipped into the tank at regular intervals.

The rest of the Factsheet will focus on continuous waves.

The wave formula

$v = \lambda \times f$

v = velocity (ms^{-1}) λ = wavelength (m) f = frequency (Hz)

To see where this formula comes from, consider how far the wave moves in one second.

- We know (from the definition of frequency) that there are f waves each second.
- Each wave is of length λ.
- So the total distance moved in one second is $f \times \lambda$
- But velocity = distance÷ time, so $v = f\lambda$

Typical Exam Question

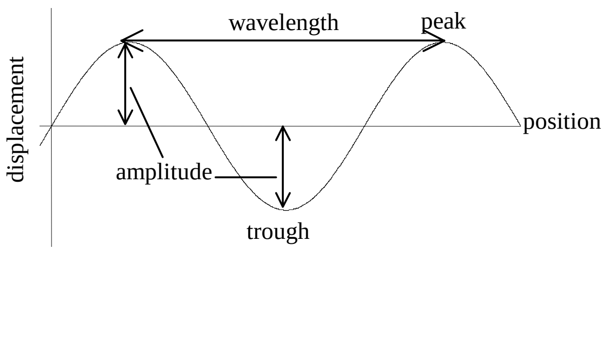
a) What is the velocity of a wave with wavelength 25cm and frequency 12 Hz? [2]
   0.25 × 12 ✓*= = 3ms^{-1}* ✓

b) A wave has a velocity of 1.25 ms^{-1}. Eight waves are observed to pass a fixed point in 2 seconds. Find
   i) the period of the wave [1]
      2÷8 = 0.25 ✓

   ii) the wavelength of the wave [2]
      f = 1/T = 4 ✓ *λ = v/f = 1.25/4 = 0.3125 m.* ✓

Glossary

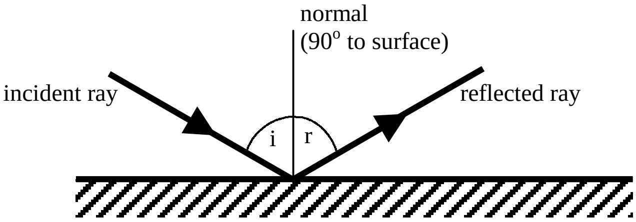
- Amplitude: the maximum displacement of a wave particle from its undisturbed position.
- Wavelenth: the distance between two similar points on a wave. Units: metres
- Frequency: the number of waves that pass a point in one second. Units: Hertz (Hz)
- Period: the time taken to complete one wave cycle. Units: second (s)
- The period (T) and the frequency (f) are related by $T = \frac{1}{f}$
- Peak(or crest): the point of maximum displacement – the “highest point”
- Trough: the point of minimum displacement – the “lowest point”



Wave properties
All waves will undergo the following processes according to the same laws:
- reflection
- refraction
- interference
- diffraction

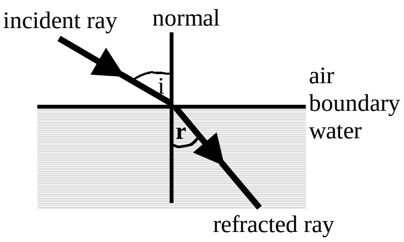
Reflection and its laws
There are two laws of reflection. They apply to both plane (flat) and curved mirrors (or other reflecting surface for waves other than light).

- the incident ray and reflected ray lie in a single plane which is perpendicular to the surface at the point of incidence
- angle of incidence (i) is equal to angle of reflection (r)

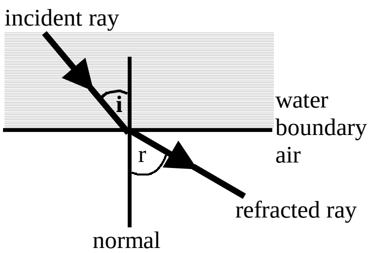


NB: The angles are **always** measured to the **normal**, **NOT** the **reflecting surface**

Refraction
Refraction is the change in direction of a wave as it crosses the boundary between two materials (eg air and water).



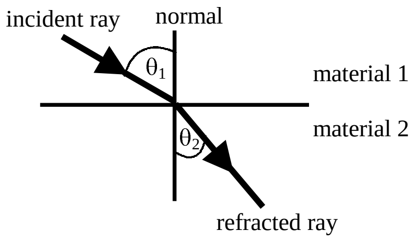
If the wave crossed the boundary in the opposite direction (i.e. water to air) then the wave direction would change in the opposite way:



Refraction is governed by two laws:

- the incident ray and reflected ray lie in a single plane which is perpendicular to the surface at the point of incidence
- "**Snell's Law**": at the boundary between any two given materials, the ratio of the sine of the angle of incidence to the sine of the angle of refraction is constant for rays of any particular wavelength.

Refractive index
Consider a wave passing from material 1 to material 2. In material 1 its angle to the normal is θ_1 and in material 2 it is θ_2.



From Snell's Law, we know that

$$\frac{\sin\theta_1}{\sin\theta_2} = \text{a constant}$$

This constant is the **refractive index of material 2 with respect to material 1**, written $_1n_2$.

So Snell's Law can be written as:

$$\frac{\sin\theta_1}{\sin\theta_2} = {}_1n_2$$

Now suppose the ray is instead travelling from material 2 to material 1. Using the above equation, we would obtain:

$$\frac{\sin\theta_2}{\sin\theta_1} = {}_2n_1$$

This gives:

$$_2n_1 = \frac{1}{_1n_2}$$

Wave speed, wavelength and refractive index
Refractive index is also related the the wave speed and wavelength in the two materials. This is used to **define the refractive index**.

$$_1n_2 = \frac{v_1}{v_2}$$

where v_1 = velocity of wave in material 1
v_2 = velocity of wave in material 2
This defines the refractive index.

Also, since **frequency does not change** during refraction:

$$_1n_2 = \frac{\lambda_1}{\lambda_2}$$

where λ_1 = wavelength of wave in material 1
λ_2 = wavelength of wave in material 2

The refractive index is often given relative to **air**. So if, for example, a question tells you that the refractive index of glass is 1.50, it means that $_{air}n_{glass} = 1.50$.

***Exam Hint**: Defining the refractive index is commonly asked. Make sure you use the equation involving wave speeds, and define all the terms.*

Typical Exam Question

The refractive index of water is *1.33*.
The speed of light in air is $3\times10^{8}\ ms^{-1}$.
a) Calculate the speed of light in water. [2]

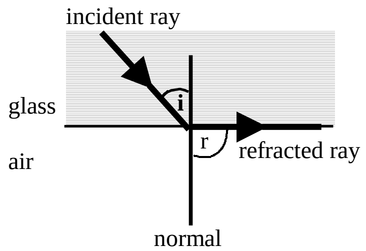
$$1.33 = \frac{v_{air}}{v_{water}} = \frac{3\times10^8}{v_{water}}\ \checkmark$$

$$v_{water} = \frac{3\times10^8}{1.33} = 2.26\times10^8\ ms^{-1}\ \checkmark$$

b) State the effect of the refraction on the frequency [1]
none – it is unchanged ✓

***Tip**: The speed of light in any other medium should always be **lower** than in vacuum (or air). Use this to check your answer – if you have got too high a speed, you have probably got the v_{air} and $v_{material}$ the wrong way up in the equation*

Critical angle and total internal reflection
When light travels from a material with a higher refractive index to one with a lower refractive index (eg from glass to air), it is possible for the angle of refraction to be 90°.



The angle at which this occurs is called the **critical angle, c**.

Using the equation $\frac{\sin\theta_1}{\sin\theta_2} = {}_1n_2$, we find: $\frac{\sin c}{\sin 90} = {}_{glass}n_{air}$

But since we are usually given ${}_{air}n_{glass}$, not ${}_{glass}n_{air}$, it is more useful to write this as:

$$\frac{\sin c}{\sin 90} = \frac{1}{{}_{air}n_{glass}}$$

Since sin90° = 1, we have:

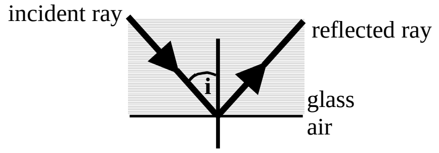
$$\sin c = \frac{1}{{}_{air}n_{glass}}$$

Given that the refractive index of glass is 1.50, we can calculate:

$$\sin c = \frac{1}{1.50} = 0.667 \Rightarrow c = 41.8^o \text{ (3 SF)}$$

Note: this cannot occur for light travelling from air to glass, since the angle to the normal decreases when travelling in this direction.

What happens for angles larger than the critical angle?
If the angle of incidence is greater than the critical angle, then the ray cannot be refracted – instead it is **totally internally reflected**



In fact, a certain amount of reflection will always occur at the interface, but for angles greater than the critical angle, **only reflection** can occur. The incident and reflected ray obey the laws of reflection.

Interference
When two sets of waves combine, we use:

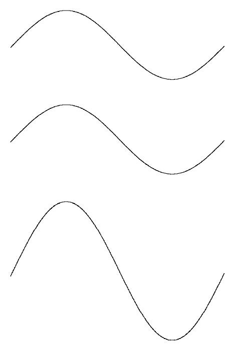
Principle of Superposition
The resultant displacement at a point is equal to the vector sum of the individual displacements at that point.

To see how this works, we will look at some examples:

1. Constructive Interference

Wave 1
+
Wave 2

Produces

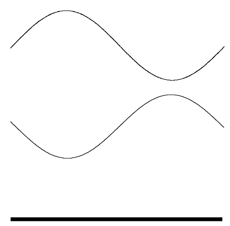


In this case, the peaks and troughs in the two waves coincide, and hence reinforce each other.

2. Destructive Interference

Wave 1
+
Wave 2

Produces

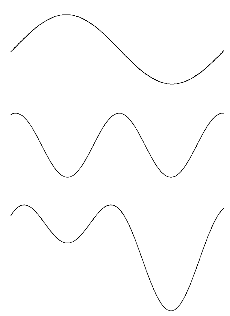


In this case, the peaks in one wave coincide with the troughs in the other, to produce no resultant displacement – the two cancel.

3.

Wave 1
+
Wave 2

Produces



In this case, peaks and troughs do not exactly coincide.

Phase difference
Phase difference is a way of measuring how far ahead one wave is of another (eg half a wavelength, quarter of a wavelength etc). It is given as an angle, with a whole wavelength corresponding to 360°. So if one wave leads another by a quarter of a wavelength, this is a phase difference of ¼ × 360° = 90°.

If the phase difference is zero (example 1 above), they are **in phase.**
If the phase difference is 180° (= half a wavelength), they are **completely out of phase** (example 2).

- Sources of waves are **coherent** if they maintain a constant phase difference and have the same frequency (eg lasers).
- A **wavefront** is a line or surface in the path of a wave motion on which all the disturbances are in phase. It is perpendicular to the direction travel of the wave.

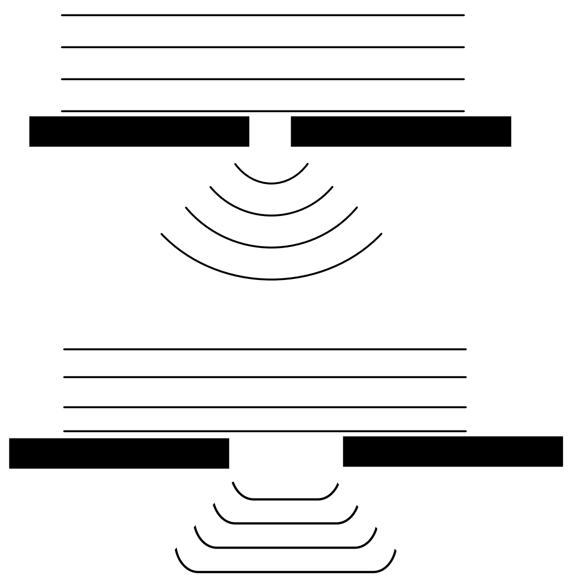
Typical Exam Question

a) Explain what is meant by superposition of waves. [2]
Waves coinciding at a point in space ✓
Disturbances add together, i.e. 'superpose' ✓

b) Distinguish between constructive and destructive interference. [4]
Constructive: waves in phase as they superpose ✓
Disturbances add to give a larger amplitude ✓
Destructive: waves 180^o out of phase ✓
Disturbances cancel to give zero amplitude ✓

c) State the conditions necessary for sources of waves to be coherent.[2]
Same frequency ✓ *Constant phase relationship* ✓

Diffraction
When waves pass an edge of an obstacle, or through a gap, they spread out and change shape. The wavelength, frequency and velocity remains constant. The extent of the spreading depends on the size of the gap, as shown below:



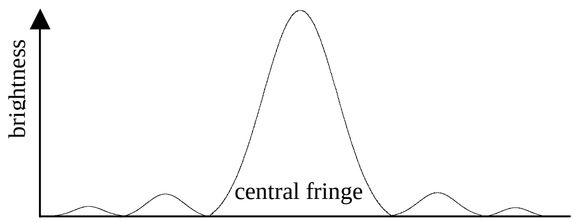
***Exam Hint**: When drawing diagrams of diffraction, make sure you keep the spacing between the wavefronts the same – this shows the frequency of the waves is unchanged.*

Appreciable diffraction only occurs if the gap is no bigger than the wavelength of the wave.

- The narrower the slit, the greater the diffraction for a particular wavelength
- The longer the wavelength for a constant slit width, the greater the diffraction

Diffraction of light and the single slit
Diffraction of light through a slit onto a screen leads to the production of light and dark fringes.

The brightness and width of the fringes can be represented graphically:

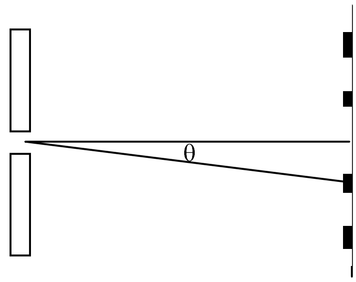


The fringe pattern has the following properties:
- It is symmetrical
- The central bright fringe is much brighter than the other fringes
- It is twice as wide as the other fringes
- The brightness (intensity of light) decreases with distance from the central fringe – so the outer fringes are the faintest.

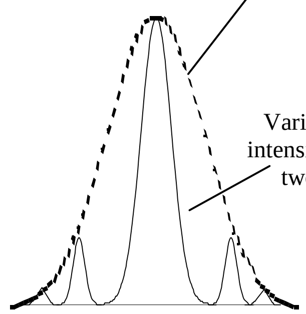
The position of the dark fringes can be calculated using:

$$\sin\theta = \frac{a\lambda}{w}$$

where θ = angle subtended in the centre (see diagram)
λ = wavelength
w = slit width
a = 1, 2, 3…. (fringe number)

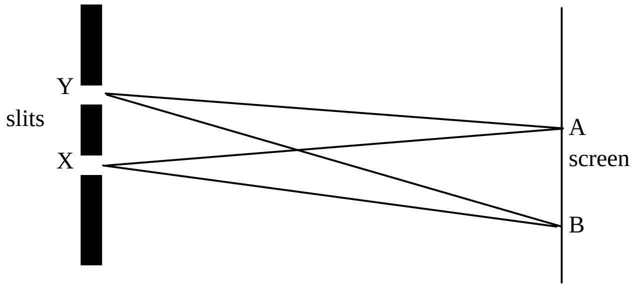


Double slit diffraction
This produces a difference in intensity within each bright fringe seen in the single slit pattern. This results from interference between light from one slit the other



This interference results from the fact that light from the different slits travels a different distance to reach a given point on the screen; this is referred to as the **path difference**.

At the centre of the screen (point A), waves from slits X and Y have travelled the same distance and therefore are in phase, and hence interfere constructively leading to a bright fringe.



As the distance from the centre changes, so does the path difference between the waves from each slit. At point B, for example, the waves from slit Y have travelled substantially further than those from slit X.

When the path difference becomes half the wavelength, then destructive interference occurs, producing a dark fringe.

Further increases in distance from the screen increases the path difference until it becomes a whole wavelength – this makes the waves in phase again, so constructive interference occurs. Further increases produce a path difference of 1½ wavelengths – giving destructive interference again.

So the bright fringes are produced from path difference $m\lambda$, and the dark fringes from path difference $(m + ½)\lambda$, where λ is the wavelength and m is any whole number.

The fringe spacing between two adjacent bright fringes is given by

$$y = \frac{D\lambda}{d}$$

D = distance to screen; λ = wavelength; d = slit spacing

In between the light and dark fringes, the interference is not perfectly constructive or destructive, so the intensity of the light changes gradually.

Polarisation

Normally, the oscillations in a transverse wave may be in many different directions. For example, for a wave travelling out of this page towards you, the oscillations will be in the plane of the page, and could be left to right, up and down, diagonally etc.

Transverse waves may undergo **polarisation**. A polarised wave oscillates in one direction only. Longitudinal waves cannot be polarised because the oscillations are already in one direction only.

> ***Exam Hint:*** *This is a key difference between longitudinal and transverse waves, and is often asked.*

Polarised light is most easily produced using a piece of Polaroid (as used in sunglasses). Polaroid works by only allowing through light which oscillates in a particular direction

If the light is passed through a vertical piece of Polaroid, then the emerging ray will be polarised vertically. It will have half the intensity of the original beam – this is why sunglasses work.

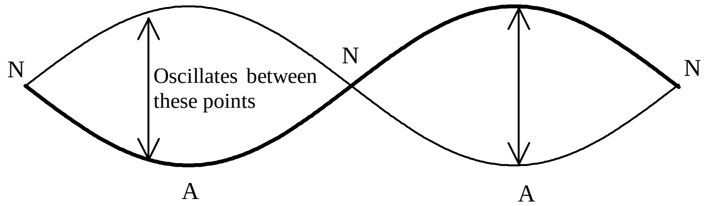
If this ray then meets a horizontal piece of Polaroid, no light will pass through.

Progressive and stationary waves

The waves discussed so far have been **progressive** i.e. they move in a particular direction, transferring energy along the direction the wave is travelling. In a **stationary** (or standing) wave, the wave does not move in a particular direction and energy is **stored** by the wave.

Stationary waves are the result of two progressive waves of the same frequency travelling in opposite directions along the same line. The diagram below shows an example of a stationary wave.

Each point on the wire can oscillate between the two positions shown.



- The **nodes** (marked N) never move. These occur where the two original waves interfere destructively
- The **antinodes** (marked A) are points of maximum displacement. They occur where the two original waves interfere constructively.

Table 1 below compares progressive and stationary waves.

Table 1. Progressive and stationary waves

Stationary wave	**Progressive wave**
Stores vibrational energy	Transmits vibrational energy
Amplitude varies	Amplitude is constant
All points between any two adjacent nodes are in phase	Phase varies smoothly with distance along the path of the wave
Nodes are half a wavelength apart; antinodes are midway between nodes	No nodes or antinodes

Typical Exam Question

a) Explain the terms node and antinode [2]
Node: point of no vibration ✓
Antinode: point of maximum vibration ✓

b) Two identical progressive waves are travelling along the same straight line in opposite directions.
(i) Explain how a stationary wave pattern is formed [3]
Stationary wave is formed by the superposition ✓ *of the two waves*
Nodes are created by destructive interference ✓
and antinodes are created by constructive interference. ✓
(ii) Compare the amplitude and phase of particles along a stationary wave with those of a progressive wave. [2]
All points on a stationary wave are in phase, points on a progressive wave are out of phase with each other ✓
All points on a progressive wave have the same amplitude, different points on stationary wave have different amplitudes ✓

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

a) (i) Explain the term 'wave front' [1]
it's at right-angles to the wave direction 0/1

> This statement is true, but does not explain wavefront. When the student read the next part of the question, s/he should have realised that this was not an adequate answer.

(ii) State the relationship between the orientation of a wave front and the direction in which the wave is travelling [1]
at right angles ✓ *1/1*

b) A longitudinal wave of frequency 30 kHz has a speed of 340ms^{-1} when travelling in air.
Its wavelength when travelling in water is 0.05m.

(i) Calculate the minimum distance between two points on the wave that differ in phase by 60^o when it is travelling through air.[3]
60^o = one sixth of wavelength ✓
0.05 ÷ 6 = 0.008m *1/3*

> *The first part of the method is correct, but the student has used the wavelength for water waves. Read the question! This shows the advantage of showing working – without it, no marks would have been awarded.*

(ii) Calculate the speed of the wave in water [2]
1.5ms^{-1} *0/2*

> The student has probably failed to convert kHz to Hz. If s/he had shown working, one mark might have been awarded, since s/he could have demonstrated the knowledge that frequency is unchanged. The answer should have worried him/her!

(iii) A pulse of the wave lasts for 10ms. Calculate the number of complete waves that it contains. [2]
30 000 × 0.01 ✓ *= 300* ✓ *2/2*

> The student has evidently now realised that the frequency is 30 kHz, not 30 Hz, but has neglected to change the earlier answer! Always make time to check.

Examiner's Answers

a) (i) A surface in which all oscillations are in phase ✓
(ii) They are perpendicular ✓

b) (i) λ = v/f
=340/30000
=0.0113m ✓
Phase difference of 60^o corresponds to λ/6 ✓
= 1.89mm ✓
(ii) speed in water = f × wavelength in water ✓
= 30 000 × 0.05 =1500ms^{-1} ✓
(iii) Number of waves = time of pulse × frequency ✓
= 0.01 × 30000 =300 ✓

Questions

1. a) Explain the difference between transverse and longitudinal waves.
   b) Give two examples of transverse waves and two examples of longitudinal waves.

2. Explain the difference between a wave pulse and a continuous wave.

3. Explain what is meant by the following terms:
   Amplitude
   Wavelength
   Peak
   Trough
   Frequency

4. Define the refractive index for a wave travelling from material 1 to material 2.

5. Explain what is meant by the Principle of Superposition.

6. Explain what is meant by diffraction.

7. Explain why sound waves cannot be polarised.

8. Give two differences between a stationary wave and a progressive wave.

9. A wave has speed 50ms^{-1} and wavelength 2m. Calculate its period.

10. The refractive index of glass is 1.50.
    a) A ray of light passes from air to glass. It makes an angle of 20^o to the normal just before entering the glass.
    Calculate the angle the refracted ray makes with the normal.
    b) The speed of light in air is 3×10^8ms^{-1}.
    Calculate the speed of light in glass.
    c) Calculate the critical angle for glass.

Answers

1. See page 1
2. See page 1
3. See page 1
4. See page 2
5. See page 2
6. See page 4
7. See page 5
8. See page 5
9. f = 50/2 = 25Hz
   T = 1/f = 0.04 s

10. a) sin20/sinr = 1.5
    sinr = sin20/1.5 = 0.228
    r = 13^o

    b) $c_{air} / c_{glass} = 1.50$
    $c_{glass} = 3 \times 10^8 / 1.50 = 2 \times 10^8$ ms^{-1}

    c) sinc = 1/1.5 = 0.667
    c = 42^o (2 SF)

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