Physics Factsheet

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Electrical Current, Voltage and Resistance

What is current?
Electricity can seem very abstract and difficult to understand. The key to grasping the subject, like so many in Physics, is to build up a picture of what is happening and follow the concepts through logically. The aim of this Factsheet is to help you do just that by explaining in simple terms what current, voltage and resistance are and how they all play a part in an electrical circuit.

An **electric current** is nothing more than a net movement or flow of charge in a certain direction. In a conducting metal the charge carriers are free electrons; these electrons originate from the rigidly bonded metal atoms that form the structure of the conductor. Their outer electrons are only weakly bonded to the atom and so many escape and are free to move throughout the structure of the metal. As the metal atoms have lost electrons they are no longer neutral but are now positively charged. Good metallic conductors include silver and copper.

Metals are not the only materials that conduct; semiconductors are a group of materials whose resistance lies somewhere between that of metallic conductors and insulators. A semiconductor is made from covalently bonded materials. Electrons in the outermost orbits only have a small 'jump' to make to move to the next orbit a little further away. When an electron does this it has two implications. As there are not many electrons in these higher levels it can jump into a vacant site in the adjacent atom and in so doing move through the element. Secondly, when it jumps up it leaves a vacant site below it known as a hole, these holes act as though they were positively charged and move the opposite way through the metal, so we get double the current we would expect. Silicon and Germanium are examples of semiconductors.

Liquids can also conduct as they long as they contain charged particles. For instance, impure water will conduct as the impurities in it exist in the form of ions, which move through the liquid. Pure water will not as the H_2O molecules are neutral.

*Current is the **rate** of **flow** of **charge***
*If the current is **constant**, we have:*

$$I = \frac{Q}{t}$$

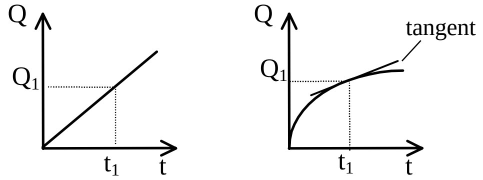
I = current (amps, A) Q = charge flowing past a point (coulombs, C)
t = time taken for the amount of charge Q to flow (seconds, s).
*This formula will also give **average current** if the current is variable.*

*If the current is **not** constant, then the gradient of a charge (y-axis) against time (x-axis) graph would give the current at a particular time*

Current and electrons
We now know that current is rate of flow of charge. The flow of charge in solids depends on the movement of charge carriers; these are generally **electrons**.

Current therefore depends on the number of free (meaning "able to move") charge carriers in the material and how quickly they move - the more charge carriers there are and the faster they move, the higher the current will be.

Finding current and average current from a graph

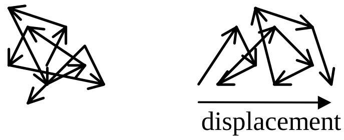


In both of the above graphs, we need to find the current at time t_1
In both cases, we use the **gradient** of the graph to find the current.

- In the first graph the gradient is constant and so therefore is the current. So dividing Q_1 by t_1 will give the right answer as you are taking the gradient of the graph.
- In example 2 dividing Q_1 by t_1 will give the **average** current up untilthat point. The gradient has been falling so the current at the time $t_1$will be less than the average value. So instead we must use a **tangent** to the line at t_1. Watch out for the distinction between average and instantaneous values.

Drift velocity
All free electrons move around due to their thermal energy even if there is no current present. However as this motion is completely random then the net effect is no overall movement. To be part of a current they have to exhibit a drift velocity in a given direction

Consider the two electron paths below:



Both display random motion as their thermal kinetic energy causes them to move, colliding with the fixed positive ions that make up a metal's structure as they go. The first electron shows very little change in displacement, the second has moved about the same distance as the first but shows a definite displacement to the right - it has a drift superimposed on its random motion. Therefore we can say the second electron is probably part of an electrical current and the first is not.

When the overall effect on all the electrons is taken into account this small drift shown by each electron provides a current, whilst when all electrons in the first example are considered the net movement of charge is zero.

Current can be calculated from:

$$I = nAQv$$

I = current (amps, I)
n = number of free charge carriers per m^3
Q: Charge on each charge carrier. (coulombs, C)
A: Cross-sectional area (m^2)
v : Drift velocity (ms^{-1})

***Exam hint**: Many students lose marks by not using the correct units in the equation above. In particular, note that cross-sectional area must be in* m^2.
To convert cm^2 *to* m^2, *divide by* $100^2 = 10\ 000$.
To convert mm^2 *to* m^2, *divide by* $1000^2 = 1000000$

If I, A and Q are constant, then v is inversely proportional to n. In other words, to carry the same current, if there are fewer charge carriers they must move at a higher speed.

Metallic conductors have a far higher value of n; for a metal and semiconductor of the same dimensions, carrying the same current, v must be higher in the semiconductor typically around m/s, as opposed to mm/s for a metal.

In an insulator there are no free charges available to carry current, therefore $n = 0m^{-3}$ and therefore, from the equation, I = 0A.

Typical Exam Question:
Calculate an average value for the drift velocity of free electrons moving through a wire of area $1.5mm^2$, when they form a current of 6.1A. Copper contains 1.0×10^{29} free electrons per m^3. The charge on an electron is 1.6×10^{-19} C **[3]**

We are going to use the equation I = nAvQ, so we need to convert the cross-sectional area into the correct units:
A in $m^2 = 1.5 \times 1 \div 10^{-6} = 1.5 \times 10^{-6} m^2$ ✓
Substitute in to our equation:
$v = I/(nAQ)$
$= 6.1/(1 \times 10^{29} \times 1.5 \times 10^{-6} \times 1.6 \times 10^{-19})$ ✓
$= 0.00025\ ms^{-1}$ ($= 0.25\ mms^{-1}$) ✓

Why does current flow?

By now you should have a picture of a conductor, for instance a metal, as structure of fixed positive ions surrounded by a sea of free electrons, colliding and rebounding with these ions as they flow, but gradually making their way from one end of the wire to the other. These electrons behave much as an incompressible liquid which explains why current starts to flow immediately when a switch is closed and the rate of flow is the same throughout, just like water flowing along a pipe.

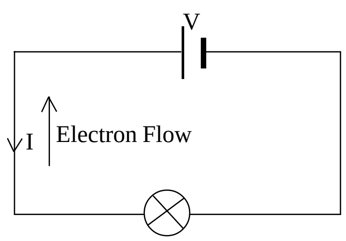
The question is; what causes charged particles to flow? The answer – **electromotive force (EMF) or voltage**. Voltage is one of most fundamental yet widely misunderstood quantities in electricity. It is a difference in potential between two points in an electrical circuit.

You can compare this to gravitational potential energy. Imagine a ball held above the ground. The ball has more potential energy because it is above the ground. It tends to fall towards the point with lower potential energy.

Similarly, a **positive** charge will "fall" from the higher (more positive) potential to the lower (more negative) potential. **Negative** charges - like **electrons** - behave in the opposite way, and move from a lower potential to a higher potential (this is where the analogy with a ball falling fails to work any more).

Current is conventionally said to be in the direction that a **positive** charge would move. So **electrons move in the opposite direction to the conventional current**.

To try and understand voltage further let's consider a simple electrical circuit.



The cell imposes an EMF on the circuit - its negative terminal contains extra electrons and its positive terminal lacks them. As soon as a complete circuit is formed the free electrons in the wire are attracted towards the positive terminal and repelled from the negative so they start to drift and are replaced by the extra electrons from the cell.

As the electrons approach the positive terminal they lose potential energy, in much the same way a mass loses potential energy as it approaches the earth. This potential energy cannot disappear - it must have been converted to a different form. In a vacuum the electrons would have gained kinetic energy as they would have accelerated towards the higher potential. However we know that electrons in a metal move with a constant (drift) velocity and therefore do not gain kinetic energy. In fact the energy is transferred to the bulb which converts it into heat and to a lesser extent light.

The light bulb does not use electrons - this is why the current either side and indeed all the way around the circuit is the same; rather it converts the loss of PE of the electrons into heat and light. This is an example of the transfer of electrical energy. This is essentially what all components in an electrical circuit are designed to do. The electrons transfer energy from the voltage source (eg. a battery) to components as they are pushed around the circuit. When a cell has lost its extra electrons and the positive terminal has become neutral it is flat and must be recharged.

Potential Difference and Electromotive Force
Although potential difference is related to energy, strictly speaking it is defined as the amount of electrical energy dissipated by a unit charge when it moves between two points in a circuit. A p.d. of 1 volt between two points means that a charge of 1 coulomb will dissipate 1 joule of energy when it moves between them.

$V = \frac{W}{Q}$

V: Potential difference between two points.
W: Energy dissipated in moving between those two points.
Q: Total charge that has moved between the two points.

An EMF is also a difference in potential but it causes the current to flow. For the EMF, W would be electrical work done on the charge by the supply (i.e. energy supplied) instead of a measure of work done by the charge on a component (i.e. energy dissipated).

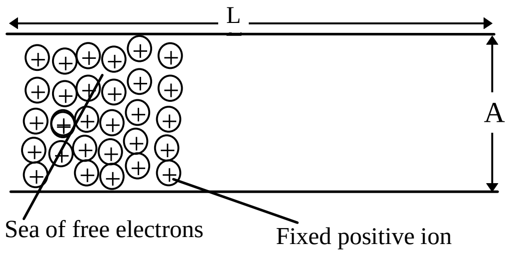
Remember p.d. is relative: two points that are at 10,000 and 9998V and two points that are at 4 and 2V both have p.d.s of 2V across them. A bird standing on a high voltage power line does not get electrocuted as both its feet are at the same, albeit very high potential. If there is no p.d. then no current flows and it survives to fly another day. When dealing with circuits we are able define one point - usually the negative terminal of the battery -as having 0V potential and then measure all other potentials relative to this.

Remember current is a flow and therefore is always **through** something. Potential difference is a difference between two points and is therefore always **across** something.

Now you should have a picture of free electrons travelling through the structure of the metal delivering energy to any component they pass through. Imagine the electrons as a traffic jam in which the traffic is nose to tail; if there is a break in the circuit no current can flow as the front electrons have nowhere to flow to, meaning electrons throughout the circuit are stationary. If the traffic moves we will get the same number of cars past every point in the road, as they all travel as fast as the cars at the front allow. In more Physical terms the sea of free electrons behaves as an incompressible liquid, with any EMF providing a push but the liquid only flows if the pipe it is travelling in is not blocked.

Resistance
Resistance in metals has its origins in the atoms that make up the material. In solid form the atoms are tightly bound into a lattice structure. It is their outermost electrons that escape to form a sea of free electrons. When the electrons escape they leave their atoms as fixed positive ions.

When we apply an emf, free electrons are pushed away from the negative terminal and towards the positive, so they try to flow from one end of the wire to the other. As they try to get past the fixed positive ions they collide repeatedly, rebounding after each collision, this generates the random component of their motion, with the applied emf imposing the drift. When they collide with the positive ions they transfer some of their kinetic energy to it. This explains why their kinetic energy does not increase as they lose potential energy - it is transferred to the wire, and also how energy is transferred to a component or wire by a current. The more the ions get in the way of the electrons, the higher the resistance and the lower the current that will flow.



If the current transfers energy to a component faster than it can dissipate the heat to the surroundings, its temperature will increase. As the fixed positive ions gain thermal KE their vibration around their fixed equilibrium positions increases and they get in the way more. The electrons find it harder to get through the metal hence the resistance increases. In all metals resistance goes up as temperature increases.

We define resistance by

$R = \frac{V}{I}$

R: Resistance in ohms (Ω)
V: potential difference across component (V)
I: current across component (A).

Ohm's law states that for a metallic conductor I α V as long as its temperature is constant, in other words if we double the voltage then the current will also double.

As a semiconductor's temperature increases more electrons are promoted so they are free to conduct. This increase in the number of charge carriers more than compensates for the increased vibrations in the material. We say it has a negative coefficient of resistance - as its temperature increases its resistance falls. This can be seen from I = nAvQ. If the number of charge carriers per unit volume increases, than the size of current for a given p.d. increases.

Resistivity

Different metals will inhibit currents by different amounts due to differences in their structures. We call this **resistivity**. We use resistivity for the same reason we use the Young Modulus - it is independent of a material's dimensions.

Using our previous diagram, if the length of the conductor is increased then its resistance must also increase as the electrons have further to travel and therefore have more ions to get past. If we increase the cross-sectional area, then resistance falls because there are more gaps for the electrons to pass through. The relationships are directly and inversely proportional respectively, i.e:

$$R \propto L/A$$

The constant of proportionality is resistivity and it is constant for a any amount of a given metal. For a conductor **it is defined as the product of resistance and cross-sectional area per unit length**.

$R = \frac{\rho L}{A}$

R: Resistance (Ω)
ρ: Resistivity(Ωm)
A: Cross-sectional area (m^2)
L: Length of conductor (m)

Remember whilst resistance depends on dimensions resisitivity, depends only on the type of metal. Sometimes you will be asked to calculate conductivity, this is just the inverse of resisitivity.

Typical Exam Question:
A heating coil made from 11 metres of wire is connected to a 240V mains supply. The wire has diameter of 0.25mm and resistivity of $1.0\times10^{-6}\Omega m$.
(a) Calculate its resistance **[4]**
(b) How much current will flow in it ? **[1]**

(a) As we are going to use R = ρl/A, we calculate the cross-sectional area:
The radius is half the diameter
r = 0.25/2 = 0.125mm ✓
$A = \pi r^2 = \pi(0.125\times10^{-3})^2 = 4.9\times10^{-8}m^2$ ✓
Now we substitute into our equation involving resistivity:
$R = \rho l/A = (1\times10^{-6}\times11)/(4.9\times10^{-8})$ ✓ $= 224\Omega$ ✓
(b) To calculate current we have a voltage and resistance:
So using I = V/ R = 240/224 = 1.07Ω ✓

Now you should be able to understand current as a flow of charged particles under the influence of a potential difference. In a metal electrons drift from one end of the wire to the other with fixed positive ions impeding their progress. It is this resistance to their motion that causes the electron's loss in potential energy to be transferred to the component the current moves through. Ohm's law links the quantities of potential difference, current and resistance and, at a constant temperature, R remains constant.

Typical Exam Question
A 12 cm length of copper wire of area $4\times10^{-7}m^2$ is connected across a potential difference of 2V. A current of 4A is measured flowing through the wire. Charge carrier density for copper = $1.0\times10^{29}m^{-3}$.
Calculate
(a) The resistivity of copper. **[4]**
(b) The drift velocity of the electrons in the wire. **[3]**
(c) If the area of the wire is doubled what effect will this have on thedrift velocity, provided the current is unchanged? **[3]**
(d) The current in the wire is increased to a point where the wire begins to heat up. What effect does this have on the resistance of the wire and why? **[3]**

(a) Firstly find the resistance using Ohm's law.
R = V/I = 2/4 = 0.5Ω ✓
Rearrange the formula and substitute in
$\rho = RA/l$ ✓ $= (0.5\times4\times10^{-7})/12 = 1.7\times10^{-8}\Omega m$ ✓
(b) Rearrange the equation containing drift velocity and substitute the values
$v = I/(nAQ)$ ✓ $= 4/(1\times10^{29}\times4\times10^{-7}m^2\times1.6\times10^{-19})$ ✓
$= 6.25\times10^{-4}ms^{-1}$ ✓
(c) As v = I/(nAQ) ✓ *and I, n and Q are constant* ✓
then if A doubles v will halve ✓*as v α 1/A.*

(d) If the wire heats up then the resistance of the wire increases ✓.
This because the fixed positive ions in the wire vibrate more ✓
and so impede the flow of electrons more ✓.

Exam Workshop

This is a typical poor student's answer to an exam question. The comments explain what is wrong with the answers and how they can be improved. The examiner's answer is given below.

(a) Define potential difference. **[2]**
Potential difference is the energy lost ✓ between two points.
1/2

> Student has forgotten about unit charge and answer should imply moving charge.

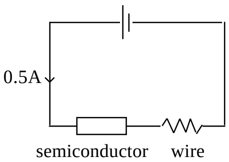
(b)i) A constantan wire has a diameter of 0.4mm and a length of 150cm. Find its resistance given the resisitivity of constantan is $5\times10^{-7}\Omega$m. **[5]**
$A = \pi r^2 = \pi \times 0.2^2$ ✓ $= 0.125m^2$ ✗
$R = \rho l/A = (5\times10^{-7}\times 1.5)/0.125$ ✗ $= 6\times10^{-6}\Omega$ ✗
1/4

> Student has remembered to halve the diameter but not to convert mm^2 to m^2. Equation is correct but the incorrect answer is obtained as area is wrong.

ii) What would happen to the resistance if the diameter and length were both doubled? **[4]**
$R = \rho l/A$
If length is doubled resistance doubles ✓
If diameter is doubled then resistance halves ✗
So overall resistance is unaffected ✗ 1/4

> Student has not realised that if the radius/diameter is doubled then the area increases by four times because we square radius to get area. Student's logic is correct with the answers worked out but does not give the correct answer.

(c) A semiconductor of the same dimensions is connected into a circuit



with the wire as shown below:
Find the ratio of drift velocities of charges in the wire compared to the semiconductor if the semiconductor has 7.2×10^{25} charge carriers per m^3 and the wire $1\times10^{29}m^{-3}$. **[3]**
For the semiconductor:
$v = I/(nAQ) = 0.5/(7.2\times10^{25}\times 0.125\times 1.6\times10^{-16}) = 3.5\times10^{-7}ms^{-1}$ ✓ecf
For the wire:
$v = I/(nAQ) = 0.5/(1\times10^{29}\times 0.125\times 1.6\times10^{-16}) = 2.5\times10^{-10}ms^{-1}$ ✓ecf
2/3

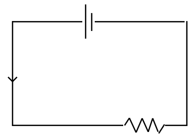
> Answer is wrong due to the area but as student has already been penalised student attains marks due too error carried forward. Student has forgotten to take a ratio.

Examiner's answers
(a) Potential difference is defined as the amount of energy transferred ✓ by unit charge ✓ when it moves between two points.
(b) i) $A = \pi r^2 = \pi\times(0.2$ ✓ $\times10^{-3}$ ✓$)^2 = 0.125\times10^{-6}m^2$ ✓
$R = \rho l/A = (5\times10^{-7}\times 1.5)/(0.125\times10^{-6})$ ✓ $= 6\Omega$ ✓
ii) $R = \rho l/A$
$R \propto$ length therefore if length is doubled resistance doubles ✓
$A \propto r^2$ therefore if diameter/radius is doubled then the area increases by four times. ✓
$R \propto 1/A$ and so resistance falls to a quarter. ✓
So overall resistance falls by one half ✓
(c) For the semiconductor
$v = I/(nAQ) = 0.5/(7.2\times10^{25}\times 0.125\times10^{-6}\times1.6\times10^{-16}) = 0.35ms^{-1}$ ✓
For the wire:
$v = I/(nAQ) = 0.5/(1x10^{29}\times0.125\times10^{-6}\times 1.6\times10^{-16}) = 2.5\times10^{-4}ms^{-1}$ ✓
$v_{semiconductor}:v_{conductor}$ 2.5×10^{-4}: 0.35 1:1400

Timed test 19 marks – 20minutes

1. (a) Define the following electrical terms and state the relevant SI units.
 i) Current [2]
 ii) Resitivity [3]
 iii) Potential difference [2]
 (b) What is the difference between the concepts of potential difference and EMF? [2]

2. (a) A p.d. of 1V is applied across the copper wire shown below.



wire of length 70cm and
cross-sectional area $7\times10^{-8}mm^2$

 i) Find its resistance. [3]
 ii) Find the current flowing in the wire. [1]
 iii) Find the drift velocity [3]

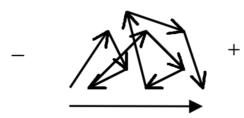
 (b) Sketch the path you would expect an electron to follow when part of a current flowing in a metallic conductor. Show the polarity of your supply. [3]

Answers

1. (a) i) Current is rate of flow of charge ✓. SI unit is the amp/ampere. ✓
 ii) Resistivity is defined as the product of resistance and area ✓ per unit length✓. SI unit ohm metre/ Ωm✓
 iii) Potential difference is the energy transferred per unit charge when a charge moves between two points in circuit ✓.
 SI unit is the volt ✓.

 (b) Emf is energy transferred to charge carriers ✓whereas potential difference is energy transferred from charge carriers to components✓.

2. (a) i) $R = \rho l/A$✓ $= 1.7\times10^{-8}\times 0.7/(7\times10^{-8})$✓ $= 0.17\Omega$ ✓
 ii) $I = V/R = 1/0.17 = 5.9A$ ✓
 iii) $v = I/(nAQ)$✓$=5.9/(1.0\times10^{29}\times7\times10^{-8}\times1.6\times10^{-19})$✓ $=5.3mms^{-1}$ ✓
 (b)



Drift element of motion shown✓
Random element of motion shown✓
Correct + and - shown.✓

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