

RMGS: MATHS ALEVEL TRANSITION

The Mathematics Department is committed to ensuring that you make good progress throughout your A level course. To help your transition from GCSE to A level we have prepared this booklet. This will help us to help you make the first steps on this journey.

The Booklet comes in 3 sections.

Section 1: Practice Worksheets

- These are topics you have seen already at GCSE.
- Please complete in your own time. **You do not have to complete all the questions.** If you already feel confident with a topic you may decide to do only some of the questions.
- We would want to see a minimum of 3 questions. (not just parts a, b, c. Three separate questions.)
- Answers are provided at the end of each section for you to check your own progress. (It is important that you mark your work to gauge an understanding of where you are)
- There is no need to print out the individual worksheets, you can read from the screen and work on a separate piece of paper
- **Please print out and complete the attached log sheet to record your progress,** we will ask to see your log sheet in September so we can see where you think your areas of Strength and Weakness are.
- Please be honest, as this information will help us help you in the best way possible,

Section 2: Baseline Assessment

- This is an assessment that should take you 1 hour to complete.
- Please do not spend any more time than that.
- **Complete on a separate sheet of paper, and show all your workings out.**
- **Try to do as much as possible without a calculator.**
- **Please ensure you complete and have ready to hand in for September.**
- We suggest taking the assessment in the week before you start back at school.
- **You will not be graded on this assessment but it will give us an idea of your starting place for A level.**

Section 3: Problem Solving

- There are a series of 10 problems in this section, all using GCSE maths techniques.
- We would like you to have a go at them.
- You may want to **tackle each problem on a separate sheet of paper.**
- We expect to see your written methods and reasoning.
- We do not want to see just an answer that has been copied from a calculator screen.
- Method and Reasoning are an important skill in A level.
- **Hand in your written solutions in September. Alongside your log book and baseline assessment.**

It is essential that you spend some time working through the questions in this booklet over the summer, like an athlete trains to improve in their events, you will need to train your maths brain so it stays active and ready to start the journey through A level.

The topics in this booklet have all been seen in some form in your GCSE and you will need to have a good knowledge of these topics before you commence your course in September.

Remember for the practice worksheets you do not have to attempt all the questions. We expect a minimum of 3 questions from each section, and your results logged on the attached record sheet. If you feel the need to give more time to one topic and complete more questions then please do so.

A-Level maths is a demanding course and good skills in the areas covered within this booklet will be paramount to your success.

We hope you have a relaxing summer and we look forward to seeing you in September.

Criteria for your ISA

BRONZE AWARD:	• A complete log book. • A completed base line assessment
SILVER AWARD:	• A complete log book. • A completed base line assessment • Up to 5 puzzles correct with written solutions.
GOLD AWARD:	• A complete log book. • A completed base line assessment • More than 5 puzzles correct with written solutions

Should you have any questions regarding the work please email:

Inistor@rmet.org (Head of Mathematics) or Ichapman@rmet.org (Key Stage Coordinator)

Log Book				
Section 1 Practice Sheets				
Please add any comments	No. Q's Attempted	Very Confident 	Some Tricky Parts 	Struggled. 
Expanding Brackets and Simplifying				
Surds				
Indices				
Factorising				
Completing the Square				
Solving Quadratic Equations				
Sketching Quadratic Graphs				
Solving Linear Simultaneous Equations				

Please add any comments	No. Q's Attempted	Very Confident 	Some Tricky Parts 	Struggled. 
Solving Simultaneous Equations Graphically.				
Linear Inequalities				
Quadratic Inequalities				
Sketching Cubic and Reciprocal Graphs				
Translating Graphs				
Straight Line Graphs				
Parallel and Perpendicular lines.				
Pythagoras Theorem				
Proportion				

Please add any comments	No. Q's Attempted	Very Confident 	Some Tricky Parts 	Struggled. 
Trigonometry				
Rearranging Equations				
Volume and Surface Area 3d Shapes				
Area under a Graph				
Section 2: Base Line Assessment				
Date Completed:				
Section 3: Problem Solving				
1: Circle Areas				
2: Fraction Multiplication				

Please add any comments		Very Confident 	Some Tricky Parts 	Struggled. 
4: Sequences				
5: Four Semi Circles				
6: How many ways to sit?				
7: Primes				
8: Sum of Indices				
9: Coordinates				
10: Inequalities.				

SECTION 1

PRACTICE WORK SHEETS

Expanding brackets and simplifying expressions

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket
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Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$ \begin{aligned} 3(x + 5) - 4(2x + 3) \\ = 3x + 15 - 8x - 12 \\ = 3 - 5x \end{aligned} $	1 Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 2 Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$
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Example 3 Expand and simplify $(x + 3)(x + 2)$

$ \begin{aligned} (x + 3)(x + 2) \\ = x(x + 2) + 3(x + 2) \\ = x^2 + 2x + 3x + 6 \\ = x^2 + 5x + 6 \end{aligned} $	1 Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 2 Simplify by collecting like terms: $2x + 3x = 5x$
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Example 4 Expand and simplify $(x - 5)(2x + 3)$

$ \begin{aligned} (x - 5)(2x + 3) \\ = x(2x + 3) - 5(2x + 3) \\ = 2x^2 + 3x - 10x - 15 \\ = 2x^2 - 7x - 15 \end{aligned} $	1 Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 2 Simplify by collecting like terms: $3x - 10x = -7x$
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Practice

- Expand.
 - $3(2x - 1)$
 - $-2(5pq + 4q^2)$
 - $-(3xy - 2y^2)$
- Expand and simplify.
 - $7(3x + 5) + 6(2x - 8)$
 - $8(5p - 2) - 3(4p + 9)$
 - $9(3s + 1) - 5(6s - 10)$
 - $2(4x - 3) - (3x + 5)$
- Expand.
 - $3x(4x + 8)$
 - $4k(5k^2 - 12)$
 - $-2h(6h^2 + 11h - 5)$
 - $-3s(4s^2 - 7s + 2)$
- Expand and simplify.
 - $3(y^2 - 8) - 4(y^2 - 5)$
 - $2x(x + 5) + 3x(x - 7)$
 - $4p(2p - 1) - 3p(5p - 2)$
 - $3b(4b - 3) - b(6b - 9)$
- Expand $\frac{1}{2}(2y - 8)$
- Expand and simplify.
 - $13 - 2(m + 7)$
 - $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle. Write down an expression, in terms of x , for the area of the rectangle.

Show that the area of the rectangle can be written as $21x^2 - 35x$

7x

8 Expand and simplify.

- $(x + 4)(x + 5)$
- $(x + 7)(x + 3)$
- $(x + 7)(x - 5)$
- $(x + 5)(x - 5)$
- $(2x + 3)(x - 1)$
- $(3x - 2)(2x + 1)$
- $(5x - 3)(2x - 5)$
- $(3x - 2)(7 + 4x)$
- $(x + 5)^2$
- $(4x - 3y)^2$
- $(2x - 7)^2$

Extend

- Expand and simplify $(x + 3)^2 + (x - 4)^2$
- Expand and simplify.
 - $\left(x + \frac{1}{x}\right)\left(x - \frac{2}{x}\right)$
 - $\left(x + \frac{1}{x}\right)^2$

Watch out!
 When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+', if the signs are different the answer is '-'.

Answers

- 1 a $6x - 3$ b $-10pq - 8q^2$
c $-3xy + 2y^2$
- 2 a $21x + 35 + 12x - 48 = 33x - 13$
b $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$
d $8x - 6 - 3x - 5 = 5x - 11$
- 3 a $12x^2 + 24x$ b $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ d $21s^2 - 21s^3 - 6s$
- 4 a $-y^2 - 4$ b $5x^2 - 11x$
c $2p - 7p^2$ d $6b^2$
- 5 $y - 4$
- 6 a $-1 - 2m$ b $5p^3 + 12p^2 + 27p$
- 7 $7x(3x - 5) = 21x^2 - 35x$
- 8 a $x^2 + 9x + 20$ b $x^2 + 10x + 21$
c $x^2 + 5x - 14$ d $x^2 - 25$
e $2x^2 + x - 3$ f $6x^2 - x - 2$
g $10x^2 - 31x + 15$ h $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ j $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ l $16x^2 - 24xy + 9y^2$
- 9 $2x^2 - 2x + 25$
- 10 a $x^2 - 1 - \frac{2}{x^2}$ b $x^2 + 2 + \frac{1}{x^2}$

Surds and rationalising the denominator

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b + \sqrt{c}}$ you multiply the numerator and denominator by $b - \sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$$\begin{aligned}\sqrt{50} &= \sqrt{25 \times 2} \\ &= \sqrt{25} \times \sqrt{2} \\ &= 5 \times \sqrt{2} \\ &= 5\sqrt{2}\end{aligned}$$

- 1 Choose two numbers that are factors of 50. One of the factors must be a square number
- 2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- 3 Use $\sqrt{25} = 5$

Example 2

Simplify $\sqrt{147} - 2\sqrt{12}$

$$\begin{aligned}\sqrt{147} - 2\sqrt{12} &= \sqrt{49 \times 3} - 2\sqrt{4 \times 3} \\ &= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3} \\ &= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3} \\ &= 7\sqrt{3} - 4\sqrt{3} \\ &= 3\sqrt{3}\end{aligned}$$

- 1 Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number
- 2 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- 3 Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
- 4 Collect like terms

Example 3

Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$$\begin{aligned}(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) &= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4} \\ &= 7 - 2 \\ &= 5\end{aligned}$$

- 1 Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$
- 2 Collect like terms:
 $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$
 $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Example 4

Rationalise $\frac{1}{\sqrt{3}}$

$$\begin{aligned}\frac{1}{\sqrt{3}} &= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\ &= \frac{1 \times \sqrt{3}}{\sqrt{9}} \\ &= \frac{\sqrt{3}}{3}\end{aligned}$$

- 1 Multiply the numerator and denominator by $\sqrt{3}$
- 2 Use $\sqrt{9} = 3$

Example 5

Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$$\begin{aligned}\frac{\sqrt{2}}{\sqrt{12}} &= \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}} \\ &= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12} \\ &= \frac{2\sqrt{2} \times \sqrt{3}}{12} \\ &= \frac{\sqrt{2} \times \sqrt{3}}{6}\end{aligned}$$

- 1 Multiply the numerator and denominator by $\sqrt{12}$
- 2 Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
- 3 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- 4 Use $\sqrt{4} = 2$
- 5 Simplify the fraction:
 $\frac{2}{12}$ simplifies to $\frac{1}{6}$

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$ $= \frac{3(2-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$ $= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$ $= \frac{6-3\sqrt{5}}{-1}$ $= 3\sqrt{5} - 6$	1 Multiply the numerator and denominator by $2-\sqrt{5}$ 2 Expand the brackets 3 Simplify the fraction 4 Divide the numerator by -1 Remember to change the sign of all terms when dividing by -1
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Practice

1 Simplify:

- a $\sqrt{45}$ b $\sqrt{125}$
c $\sqrt{48}$ d $\sqrt{175}$
e $\sqrt{300}$ f $\sqrt{28}$
g $\sqrt{72}$ h $\sqrt{162}$

Hint

One of the two numbers you choose at the start must be a square number.

2 Simplify:

- a $\sqrt{72} + \sqrt{162}$ b $\sqrt{45} - 2\sqrt{5}$
c $\sqrt{50} - \sqrt{8}$ d $\sqrt{75} - \sqrt{48}$
e $2\sqrt{28} + \sqrt{28}$ f $2\sqrt{12} - \sqrt{12} + \sqrt{27}$

Watch out!

Check you have chosen the highest square number at the start.

3 Expand and simplify:

- a $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$ b $(3 + \sqrt{3})(5 - \sqrt{12})$
c $(4 - \sqrt{5})(\sqrt{45} + 2)$ d $(5 + \sqrt{2})(6 - \sqrt{8})$

4 Rationalise and simplify, if possible.

- a $\frac{1}{\sqrt{5}}$ b $\frac{1}{\sqrt{11}}$
c $\frac{2}{\sqrt{7}}$ d $\frac{2}{\sqrt{8}}$
e $\frac{2}{\sqrt{2}}$ f $\frac{5}{\sqrt{5}}$
g $\frac{\sqrt{8}}{\sqrt{24}}$ h $\frac{\sqrt{5}}{\sqrt{45}}$
a $\frac{1}{3-\sqrt{5}}$ b $\frac{2}{4+\sqrt{3}}$ c $\frac{6}{5-\sqrt{2}}$

5 Rationalise and simplify:

6 Expand and simplify $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

7 Rationalise and simplify, if possible.

- a $\frac{1}{\sqrt{9} - \sqrt{8}}$ b $\frac{1}{\sqrt{x} - \sqrt{y}}$

Extend

Answers

1 a $3\sqrt{5}$

c $4\sqrt{3}$

e $10\sqrt{3}$

g $6\sqrt{2}$

2 a $15\sqrt{2}$

c $3\sqrt{2}$

e $6\sqrt{7}$

3 a -1

c $10\sqrt{5}-7$

4 a $\frac{\sqrt{5}}{5}$

c $\frac{2\sqrt{7}}{7}$

e $\sqrt{2}$

g $\frac{\sqrt{3}}{3}$

5 a $\frac{3+\sqrt{5}}{4}$

6 $x-y$

7 a $3+2\sqrt{2}$

b $5\sqrt{5}$

d $5\sqrt{7}$

f $2\sqrt{7}$

h $9\sqrt{2}$

b $\sqrt{5}$

d $\sqrt{3}$

f $5\sqrt{3}$

b $9-\sqrt{3}$

d $26-4\sqrt{2}$

b $\frac{\sqrt{11}}{11}$

d $\frac{\sqrt{2}}{2}$

f $\sqrt{5}$

h $\frac{1}{3}$

b $\frac{2(4-\sqrt{3})}{13}$

c $\frac{6(5+\sqrt{2})}{23}$

b $\frac{\sqrt{x}+\sqrt{y}}{x-y}$

Rules of indices

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1
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Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9} = 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
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Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9$	1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use $\sqrt[3]{27} = 3$
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Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$	1 Use the rule $a^{-m} = \frac{1}{a^m}$ 2 Use $4^2 = 16$
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Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
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Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4} = x^{8-4} = x^4$	1 Use the rule $a^m \times a^n = a^{m+n}$ 2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$
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Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged
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Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}} = 4x^{-\frac{1}{2}}$	1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use the rule $\frac{1}{a^m} = a^{-m}$
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Practice

1 Evaluate.

a 14^0

2 Evaluate.

a $49^{\frac{1}{2}}$

3 Evaluate.

a $25^{\frac{3}{2}}$

4 Evaluate.

a 5^{-2}

5 Simplify.

a $\frac{3x^2 \times x^3}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$

e $\frac{y^2}{y^{\frac{1}{3}} \times y}$

g $\frac{(2x^2)^3}{4x^0}$

6 Evaluate.

a $4^{\frac{1}{2}}$

d $16^{\frac{1}{4}} \times 2^{-3}$

7 Write the following as a single power of x .

a $\frac{1}{x}$

d $\sqrt[4]{x^2}$

b 3^0

b $64^{\frac{1}{3}}$

b $8^{\frac{5}{3}}$

b 4^{-3}

b $\frac{10x^5}{2x^2 \times x}$

d $\frac{7x^3y^2}{14x^2y}$

f $\frac{c^{\frac{1}{3}}}{c^2 \times c^{\frac{2}{3}}}$

h $\frac{x^{\frac{1}{3}} \times x^{\frac{2}{3}}}{x^2 \times x^3}$

b $27^{-\frac{2}{3}}$

e $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

b $\frac{1}{x^{\frac{1}{2}}}$

e $\frac{1}{\sqrt[3]{x}}$

d x^0

d $16^{\frac{1}{4}}$

d $16^{\frac{3}{4}}$

d 6^{-2}

Watch out!
Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

8 Write the following without negative or fractional powers.

a x^{-3} b x^0 c $\frac{1}{x^5}$
d $\frac{2}{x^5}$ e $\frac{1}{x^{\frac{1}{2}}}$ f $\frac{3}{x^{\frac{1}{4}}}$

9 Write the following in the form ax^n .

a $5\sqrt{x}$ b $\frac{2}{x^3}$ c $\frac{1}{3x^4}$
d $\frac{2}{\sqrt{x}}$ e $\frac{4}{\sqrt[3]{x}}$ f 3

Extend

10 Write as sums of powers of x .

a $\frac{x^5+1}{x^2}$ b $x^2\left(x+\frac{1}{x}\right)$ c $x^{-4}\left(x^2+\frac{1}{x^2}\right)$

Answers

- 1 a 1 b 1 c 1 d 1
- 2 a 7 b 4 c 5 d 2
- 3 a 125 b 32 c 343 d 8
- 4 a $\frac{1}{25}$ b $\frac{1}{64}$ c $\frac{1}{32}$ d $\frac{1}{36}$
- 5 a $\frac{3x^3}{2}$ b $5x^2$ c $3x$ d $\frac{y}{2x^2}$
- e y^2 f c^3 g $2x^6$ h x
- 6 a $\frac{1}{2}$ b $\frac{1}{9}$ c $\frac{8}{3}$ d $\frac{1}{4}$ e $\frac{4}{3}$ f $\frac{16}{9}$
- 7 a x^{-1} b x^{-7} c $\frac{1}{x^4}$ d $\frac{2}{x^5}$ e $\frac{1}{x^3}$ f $\frac{2}{x^3}$
- 8 a $\frac{1}{x^3}$ b 1 c $\sqrt[3]{x}$ d $\frac{1}{\sqrt[3]{x^2}}$ e $\frac{1}{\sqrt{x}}$ f $\frac{1}{\sqrt[3]{x^3}}$
- 9 a $\frac{1}{5x^2}$ b $2x^3$ c $\frac{1}{3}x^{-4}$ d $\frac{1}{2x^2}$ e $4x^{\frac{1}{3}}$ f $3x^0$
- 10 a $x^3 + x^{-2}$ b $x^3 + x$ c $x^{-2} + x^{-7}$

Factorising expressions

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is c .
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$$

The highest common factor is $3x^2y$.
So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$$

This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$$\begin{aligned} b &= 3, ac = -10 \\ \text{So } x^2 + 3x - 10 &= x^2 + 5x - 2x - 10 \\ &= x(x + 5) - 2(x + 5) \\ &= (x + 5)(x - 2) \end{aligned}$$

- Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2)
- Rewrite the b term ($3x$) using these two factors
- Factorise the first two terms and the last two terms
- $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$$\begin{aligned} b &= -11, ac = -60 \\ \text{So } 6x^2 - 11x - 10 &= 6x^2 - 15x + 4x - 10 \\ &= 3x(2x - 5) + 2(2x - 5) \\ &= (2x - 5)(3x + 2) \end{aligned}$$

- Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4)
- Rewrite the b term ($-11x$) using these two factors
- Factorise the first two terms and the last two terms
- $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$$\begin{aligned} \frac{x^2 - 4x - 21}{2x^2 + 9x + 9} \\ \text{For the numerator: } b &= -4, ac = -21 \\ \text{So } x^2 - 4x - 21 &= x^2 - 7x + 3x - 21 \\ &= x(x - 7) + 3(x - 7) \\ &= (x - 7)(x + 3) \\ \text{For the denominator: } b &= 9, ac = 18 \\ \text{So } 2x^2 + 9x + 9 &= 2x^2 + 6x + 3x + 9 \\ &= 2x(x + 3) + 3(x + 3) \\ &= (x + 3)(2x + 3) \\ \text{So } \frac{x^2 - 4x - 21}{2x^2 + 9x + 9} &= \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)} \\ &= \frac{x - 7}{2x + 3} \end{aligned}$$

- Factorise the numerator and the denominator
- Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
- Rewrite the b term ($-4x$) using these two factors
- Factorise the first two terms and the last two terms
- $(x - 7)$ is a factor of both terms
- Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
- Rewrite the b term ($9x$) using these two factors
- Factorise the first two terms and the last two terms
- $(x + 3)$ is a factor of both terms
- $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice

- Factorise.
 - $6x^4y^3 - 10x^3y^4$
 - $25x^2y^2 - 10x^3y^2 + 15x^2y^3$
- Factorise
 - $x^2 + 7x + 12$
 - $x^2 - 11x + 30$
 - $x^2 - 7x - 18$
 - $x^2 - 3x - 40$
- Factorise
 - $36x^2 - 49y^2$
 - $18a^2 - 200b^2c^2$
- Factorise
 - $2x^2 + x - 3$
 - $2x^2 + 7x + 3$
 - $10x^2 + 21x + 9$
- Simplify the algebraic fractions.
 - $\frac{2x^2 + 4x}{x^2 - x}$
 - $\frac{x^2 - 2x - 8}{x^2 - 4x}$
 - $\frac{x^2 - x - 12}{x^2 - 4x}$
- Simplify
 - $\frac{9x^2 - 16}{3x^2 + 17x - 28}$
 - $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

Hint

Take the highest common factor outside the bracket.

- $21a^3b^5 + 35a^5b^2$
- $x^2 + 5x - 14$
- $x^2 - 5x - 24$
- $x^2 + x - 20$
- $x^2 + 3x - 28$
- $4x^2 - 81y^2$
- $6x^2 + 17x + 5$
- $9x^2 - 15x + 4$
- $12x^2 - 38x + 20$
- $\frac{x^2 + 3x}{x^2 + 2x - 3}$
- $\frac{x^2 - 5x}{x^2 - 25}$
- $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$
- $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$
- $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

- Simplify $\sqrt{x^2 + 10x + 25}$
- Simplify $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Answers

- $2x^3y^3(3x - 5y)$
 - $5x^2y^2(5 - 2x + 3y)$
- $(x+3)(x+4)$
 - $(x-5)(x-6)$
 - $(x-9)(x+2)$
 - $(x-8)(x+5)$
- $(6x-7y)(6x+7y)$
 - $2(3a-10bc)(3a+10bc)$
- $(x-1)(2x+3)$
 - $(2x+1)(x+3)$
 - $(5x+3)(2x+3)$
- $\frac{2(x+2)}{x-1}$
 - $\frac{x+2}{x}$
 - $\frac{x+3}{x}$
- $\frac{3x+4}{x+7}$
 - $\frac{2-5x}{2x-3}$
- $(x+5)$
- $\frac{4(x+2)}{x-2}$

- $7a^3b^2(3b^3 + 5a^2)$
- $(x+7)(x-2)$
- $(x-8)(x+3)$
- $(x+5)(x-4)$
- $(x+7)(x-4)$
- $(2x-9y)(2x+9y)$
- $(3x+1)(2x+5)$
- $(3x-1)(3x-4)$
- $2(3x-2)(2x-5)$
- $\frac{x}{x-1}$
- $\frac{x}{x+5}$
- $\frac{x}{x-5}$
- $\frac{2x+3}{3x-2}$
- $\frac{3x+1}{x+4}$

Completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	1 Write $x^2 + 6x + c$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ 2 Simplify
---	---

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ 2 Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ 3 Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2 4 Simplify
--	---

Practice

- Write the following quadratic expressions in the form $(x + p)^2 + q$
 - $x^2 + 4x + 3$
 - $x^2 - 10x - 3$
 - $x^2 - 8x$
 - $x^2 + 6x$
 - $x^2 - 2x + 7$
 - $x^2 + 3x - 2$
- Write the following quadratic expressions in the form $p(x + q)^2 + r$
 - $2x^2 - 8x - 16$
 - $4x^2 - 8x - 16$
 - $3x^2 + 12x - 9$
 - $2x^2 + 6x - 8$
- Complete the square.
 - $2x^2 + 3x + 6$
 - $3x^2 - 2x$
 - $5x^2 + 3x$
 - $3x^2 + 5x + 3$

Extend

- Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

- 1 a $(x+2)^2 - 1$ b $(x-5)^2 - 28$
c $(x-4)^2 - 16$ d $(x+3)^2 - 9$
e $(x-1)^2 + 6$ f $\left(x + \frac{3}{2}\right)^2 - \frac{17}{4}$
- 2 a $2(x-2)^2 - 24$ b $4(x-1)^2 - 20$
c $3(x+2)^2 - 21$ d $2\left(x + \frac{3}{2}\right)^2 - \frac{25}{2}$
- 3 a $2\left(x + \frac{3}{4}\right)^2 + \frac{39}{8}$ b $3\left(x - \frac{1}{3}\right)^2 - \frac{1}{3}$
c $5\left(x + \frac{3}{10}\right)^2 - \frac{9}{20}$ d $3\left(x + \frac{5}{6}\right)^2 + \frac{11}{12}$
- 4 $(5x+3)^2 + 3$

Solving quadratic equations by factorisation

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac .
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$	1 Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x^2 - 15x = 0$	2 Factorise the quadratic equation. $5x$ is a common factor.
$5x(x - 3) = 0$	3 When two values multiply to make zero, at least one of the values must be zero.
So $5x = 0$ or $(x - 3) = 0$	4 Solve these two equations.
Therefore $x = 0$ or $x = 3$	

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$	1 Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$b = 7, ac = 12$	2 Rewrite the b term ($7x$) using these two factors.
$x^2 + 4x + 3x + 12 = 0$	3 Factorise the first two terms and the last two terms.
$x(x + 4) + 3(x + 4) = 0$	4 ($x + 4$) is a factor of both terms.
$(x + 4)(x + 3) = 0$	5 When two values multiply to make zero, at least one of the values must be zero.
So $(x + 4) = 0$ or $(x + 3) = 0$	6 Solve these two equations.
Therefore $x = -4$ or $x = -3$	

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$	1 Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$.
$(3x + 4)(3x - 4) = 0$	2 When two values multiply to make zero, at least one of the values must be zero.
So $(3x + 4) = 0$ or $(3x - 4) = 0$	3 Solve these two equations.
$x = -\frac{4}{3}$ or $x = \frac{4}{3}$	

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5, ac = -24$	1 Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3)
So $2x^2 - 8x + 3x - 12 = 0$	2 Rewrite the b term ($-5x$) using these two factors.
$2x(x - 4) + 3(x - 4) = 0$	3 Factorise the first two terms and the last two terms.
$(x - 4)(2x + 3) = 0$	4 ($x - 4$) is a factor of both terms.
So $(x - 4) = 0$ or $(2x + 3) = 0$	5 When two values multiply to make zero, at least one of the values must be zero.
$x = 4$ or $x = -\frac{3}{2}$	6 Solve these two equations.

Practice

1 Solve

- a** $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$
c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$
e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$
g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$
i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$
k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

- a** $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$
c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$
e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$
g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by completing the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x+q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	1 Write $x^2 + bx + c = 0$ in the form $\left(x + \frac{b}{2}\right)^2 + c = 0$
$(x+3)^2 - 9 + 4 = 0$	2 Simplify.
$(x+3)^2 - 5 = 0$	3 Rearrange the equation to work out x . First, add 5 to both sides.
$(x+3)^2 = 5$	4 Square root both sides. Remember that the square root of a value gives two answers.
$x+3 = \pm\sqrt{5}$	5 Subtract 3 from both sides to solve the equation.
$x = \pm\sqrt{5} - 3$	6 Write down both solutions.
So $x = -\sqrt{5} - 3$ or $x = \sqrt{5} - 3$	

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	2 Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	3 Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	4 Simplify.
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	

(continued on next page)

$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	5 Rearrange the equation to work out x . First, add $\frac{17}{8}$ to both sides.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	6 Divide both sides by 2.
$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$	7 Square root both sides. Remember that the square root of a value gives two answers.
$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$	8 Add $\frac{7}{4}$ to both sides.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	9 Write down both the solutions.

Practice

3 Solve by completing the square.

- a $x^2 - 4x - 3 = 0$ b $x^2 - 10x + 4 = 0$
c $x^2 + 8x - 5 = 0$ d $x^2 - 2x - 6 = 0$
e $2x^2 + 8x - 5 = 0$ f $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

- a $(x-4)(x+2) = 5$
b $2x^2 + 6x - 7 = 0$
c $x^2 - 5x + 3 = 0$

Hint

Get all terms onto one side of the equation.

Solving quadratic equations by using the formula

A LEVEL LINKS

Scheme of work: 11b. Quadratic functions — factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a , b and c .

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1, b = 6, c = 4$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	1 Identify a , b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	2 Substitute $a = 1, b = 6, c = 4$ into the formula.
$x = \frac{-6 \pm \sqrt{20}}{2}$	3 Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \frac{-6 \pm 2\sqrt{5}}{2}$	4 Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	5 Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	6 Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3, b = -7, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	1 Identify a , b and c , making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	2 Substitute $a = 3, b = -7, c = -2$ into the formula.
$x = \frac{7 \pm \sqrt{73}}{6}$	3 Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	4 Write down both the solutions.

Practice

- 5** Solve, giving your solutions in surd form.
- a** $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$
- 6** Solve the equation $x^2 - 7x + 2 = 0$
Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a , b and c are integers.
- 7** Solve $10x^2 + 3x + 3 = 5$
Give your solution in surd form.

Hint

Get all terms onto one side of the equation.

Extend

- 8** Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.
- a** $4x(x - 1) = 3x - 2$
b $10 = (x + 1)^2$
c $x(3x - 1) = 10$

Answers

- 1 a $x = 0$ or $x = \frac{2}{3}$ b $x = 0$ or $x = \frac{3}{4}$
c $x = -5$ or $x = -2$ d $x = 2$ or $x = 3$
e $x = -1$ or $x = 4$ f $x = -5$ or $x = 2$
g $x = 4$ or $x = 6$ h $x = -6$ or $x = 6$
i $x = -7$ or $x = 4$ j $x = 3$
k $x = -\frac{1}{2}$ or $x = 4$ l $x = -\frac{2}{3}$ or $x = 5$
- 2 a $x = -2$ or $x = 5$ b $x = -1$ or $x = 3$
c $x = -8$ or $x = 3$ d $x = -6$ or $x = 7$
e $x = -5$ or $x = 5$ f $x = -4$ or $x = 7$
g $x = -3$ or $x = 2\frac{1}{2}$ h $x = -\frac{1}{3}$ or $x = 2$
- 3 a $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ b $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$
c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ d $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$
e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ f $x = \frac{-3 + \sqrt{89}}{10}$ or $x = \frac{-3 - \sqrt{89}}{10}$
- 4 a $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ b $x = \frac{-3 + \sqrt{23}}{2}$ or $x = \frac{-3 - \sqrt{23}}{2}$
c $x = \frac{5 + \sqrt{13}}{2}$ or $x = \frac{5 - \sqrt{13}}{2}$
- 5 a $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ b $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$
- 6 $x = \frac{7 + \sqrt{41}}{2}$ or $x = \frac{7 - \sqrt{41}}{2}$
- 7 $x = \frac{-3 + \sqrt{89}}{20}$ or $x = \frac{-3 - \sqrt{89}}{20}$
- 8 a $x = \frac{7 + \sqrt{17}}{8}$ or $x = \frac{7 - \sqrt{17}}{8}$
b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$
c $x = -1\frac{2}{3}$ or $x = 2$

Sketching quadratic graphs

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

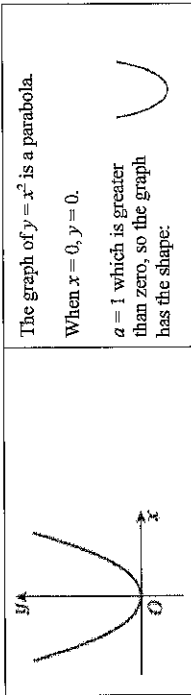
Key points

- The graph of the quadratic function $y = ax^2 + bx + c$, where $a \neq 0$, is a curve called a parabola.
- Parabolas have a line of symmetry and a shape as shown.
- To sketch the graph of a function, find the points where the graph intersects the axes.
- To find where the curve intersects the y -axis substitute $x = 0$ into the function.
- To find where the curve intersects the x -axis substitute $y = 0$ into the function.
- At the turning points of a graph the gradient of the curve is 0 and any tangents to the curve at these points are horizontal.
- To find the coordinates of the maximum or minimum point (turning points) of a quadratic curve (parabola) you can use the completed square form of the function.

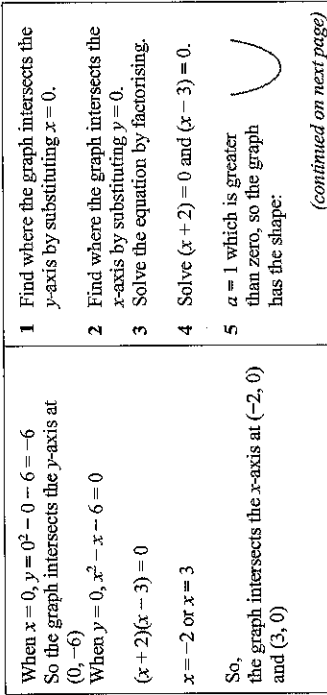


Examples

Example 1 Sketch the graph of $y = x^2$.



Example 2 Sketch the graph of $y = x^2 - x - 6$.



6 To find the turning point, complete the square.

$$x^2 - x - 6 = \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 6$$

$$= \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$$

When $\left(x - \frac{1}{2}\right)^2 = 0$, $x = \frac{1}{2}$ and $y = -\frac{25}{4}$, so the turning point is at the point $\left(\frac{1}{2}, -\frac{25}{4}\right)$

7 The turning point is the minimum value for this expression and occurs when the term in the bracket is equal to zero.

Practice

- Sketch the graph of $y = -x^2$.
- Sketch each graph, labelling where the curve crosses the axes.

a $y = (x + 2)(x - 1)$

b $y = x(x - 3)$

c $y = (x + 1)(x + 5)$
- Sketch each graph, labelling where the curve crosses the axes.

a $y = x^2 - x - 6$

b $y = x^2 - 5x + 4$

c $y = x^2 - 4$

d $y = x^2 + 4x$

e $y = 9 - x^2$

f $y = x^2 + 2x - 3$
- Sketch the graph of $y = 2x^2 + 5x - 3$, labelling where the curve crosses the axes.

Extend

- Sketch each graph. Label where the curve crosses the axes and write down the coordinates of the turning point.

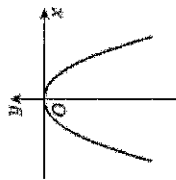
a $y = x^2 - 5x + 6$

b $y = -x^2 + 7x - 12$

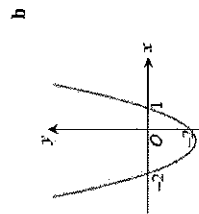
c $y = -x^2 + 4x$
- Sketch the graph of $y = x^2 + 2x + 1$. Label where the curve crosses the axes and write down the equation of the line of symmetry.

Answers

1



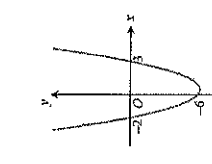
2



b

c

3



b

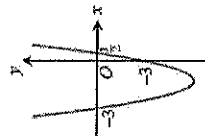
c

d

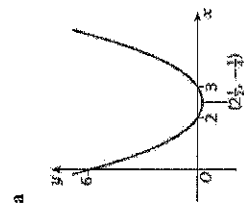
e

f

4



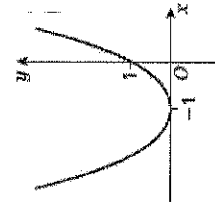
5



b

c

6



Line of symmetry at $x = -1$.

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x = 4 \\ \text{So } x = 2 \end{array}$ Using $x + y = 1$ $2 + y = 1$ So $y = -1$ Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	1 Subtract the second equation from the first equation to eliminate the y term. 2 To find the value of y, substitute $x = 2$ into one of the original equations. 3 Substitute the values of x and y into both equations to check your answers.
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Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + \quad 5x - 2y = 5 \\ \hline 6x = 18 \\ \text{So } x = 3 \end{array}$ Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$ Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	1 Add the two equations together to eliminate the y term. 2 To find the value of y, substitute $x = 3$ into one of the original equations. 3 Substitute the values of x and y into both equations to check your answers.
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Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$\begin{array}{r} (2x + 3y = 2) \times 4 \rightarrow 8x + 12y = 8 \\ (5x + 4y = 12) \times 3 \rightarrow 15x + 12y = 36 \\ \hline 7x = 28 \\ \text{So } x = 4 \end{array}$ Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$ Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	1 Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term. 2 To find the value of x, substitute $x = 4$ into one of the original equations. 3 Substitute the values of x and y into both equations to check your answers.
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Practice

Solve these simultaneous equations.

- | | |
|--|--|
| 1 $4x + y = 8$
$x + y = 5$ | 2 $3x + y = 7$
$3x + 2y = 5$ |
| 3 $4x + y = 3$
$3x - y = 11$ | 4 $3x + 4y = 7$
$x - 4y = 5$ |
| 5 $2x + y = 11$
$x - 3y = 9$ | 6 $2x + 3y = 11$
$3x + 2y = 4$ |

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous
Textbook: Pure Year 1, 3.1. Linear simultaneous equations

Key points

- The substitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	1 Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$	2 Expand the brackets and simplify.
$11x + 3 = 14$	3 Work out the value of x .
$11x = 11$	
So $x = 1$	
Using $y = 2x + 1$	4 To find the value of y , substitute $x = 1$ into one of the original equations.
$y = 2 \times 1 + 1$	
So $y = 3$	
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	5 Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$	1 Rearrange the first equation.
$4x + 3(2x - 16) = -3$	2 Substitute $2x - 16$ for y into the second equation.
$4x + 6x - 48 = -3$	3 Expand the brackets and simplify.
$10x - 48 = -3$	4 Work out the value of x .
$10x = 45$	
So $x = 4\frac{1}{2}$	
Using $y = 2x - 16$	5 To find the value of y , substitute $x = 4\frac{1}{2}$ into one of the original equations.
$y = 2 \times 4\frac{1}{2} - 16$	
So $y = -7$	
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	6 Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

- | | |
|---|---|
| 7 $y = x - 4$
$2x + 5y = 43$ | 8 $y = 2x - 3$
$5x - 3y = 11$ |
| 9 $2y = 4x + 5$
$9x + 5y = 22$ | 10 $2x = y - 2$
$8x - 5y = -11$ |
| 11 $3x + 4y = 8$
$2x - y = -13$ | 12 $3y = 4x - 7$
$2y = 3x - 4$ |
| 13 $3x = y - 1$
$2y - 2x = 3$ | 14 $3x + 2y + 1 = 0$
$4y = 8 - x$ |

Extend

- 15** Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x + y) = \frac{3(y - x)}{4}$.

Answers

- 1 $x=1, y=4$
- 2 $x=3, y=-2$
- 3 $x=2, y=-5$
- 4 $x=3, y=-\frac{1}{2}$
- 5 $x=6, y=-1$
- 6 $x=-2, y=5$
- 7 $x=9, y=5$
- 8 $x=-2, y=-7$
- 9 $x=\frac{1}{2}, y=3\frac{1}{2}$
- 10 $x=\frac{1}{2}, y=3$
- 11 $x=-4, y=5$
- 12 $x=-2, y=-5$
- 13 $x=\frac{1}{4}, y=1\frac{3}{4}$
- 14 $x=-2, y=2\frac{1}{2}$
- 15 $x=-2\frac{1}{2}, y=5\frac{1}{2}$

Solving linear and quadratic simultaneous equations

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Make one of the unknowns the subject of the linear equation (rearranging where necessary).
- Use the linear equation to substitute into the quadratic equation.
- There are usually two pairs of solutions.

Examples

Example 1 Solve the simultaneous equations $y = x + 1$ and $x^2 + y^2 = 13$

$x^2 + (x+1)^2 = 13$ $x^2 + x^2 + x + x + 1 = 13$ $2x^2 + 2x + 1 = 13$	1 Substitute $x + 1$ for y into the second equation.
$2x^2 + 2x - 12 = 0$ $(2x - 4)(x + 3) = 0$ So $x = 2$ or $x = -3$	2 Expand the brackets and simplify.
Using $y = x + 1$ When $x = 2, y = 2 + 1 = 3$ When $x = -3, y = -3 + 1 = -2$	3 Factorise the quadratic equation.
So the solutions are $x = 2, y = 3$ and $x = -3, y = -2$	4 Work out the values of x .
Check: equation 1: $3 = 2 + 1$ YES and $-2 = -3 + 1$ YES equation 2: $2^2 + 3^2 = 13$ YES and $(-3)^2 + (-2)^2 = 13$ YES	5 To find the value of y , substitute both values of x into one of the original equations.
	6 Substitute both pairs of values of x and y into both equations to check your answers.

Example 2 Solve $2x + 3y = 5$ and $2y^2 + xy = 12$ simultaneously.

$x = \frac{5-3y}{2}$ $2y^2 + \left(\frac{5-3y}{2}\right)y = 12$ $2y^2 + \frac{5y-3y^2}{2} = 12$ $4y^2 + 5y - 3y^2 = 24$ $y^2 + 5y - 24 = 0$ $(y+8)(y-3) = 0$ So $y = -8$ or $y = 3$	1 Rearrange the first equation.
Using $2x + 3y = 5$ When $y = -8, 2x + 3 \times (-8) = 5, x = 14.5$ When $y = 3, 2x + 3 \times 3 = 5, x = -2$	2 Substitute $\frac{5-3y}{2}$ for x into the second equation. Notice how it is easier to substitute for x than for y .
So the solutions are $x = 14.5, y = -8$ and $x = -2, y = 3$	3 Expand the brackets and simplify.
Check: equation 1: $2 \times 14.5 + 3 \times (-8) = 5$ YES and $2 \times (-2) + 3 \times 3 = 5$ YES equation 2: $2 \times (-8)^2 + 14.5 \times (-8) = 12$ YES and $2 \times (3)^2 + (-2) \times 3 = 12$ YES	4 Factorise the quadratic equation.
	5 Work out the values of y .
	6 To find the value of x , substitute both values of y into one of the original equations.
	7 Substitute both pairs of values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

- $y = 2x + 1$
 $x^2 + y^2 = 10$
- $y = 6 - x$
 $x^2 + y^2 = 20$
- $y = x - 3$
 $x^2 + y^2 = 5$
- $y = 9 - 2x$
 $x^2 + y^2 = 17$
- $y = 3x - 5$
 $y = x^2 - 2x + 1$
- $y = x - 5$
 $y = x^2 - 5x - 12$
- $y = x + 5$
 $x^2 + y^2 = 25$
- $y = 2x - 1$
 $x^2 + xy = 24$
- $y = 2x$
 $y^2 - xy = 8$
- $2x + y = 11$
 $xy = 15$
- $y - x = 2$
 $x^2 + y^2 = 3$

Extend

- $x - y = 1$
 $x^2 + y^2 = 3$
- $y - x = 2$
 $x^2 + y^2 = 3$

Answers

1 $x = 1, y = 3$
 $x = -\frac{9}{5}, y = -\frac{13}{5}$

2 $x = 2, y = 4$
 $x = 4, y = 2$

3 $x = 1, y = -2$
 $x = 2, y = -1$

4 $x = 4, y = 1$
 $x = \frac{16}{5}, y = \frac{13}{5}$

5 $x = 3, y = 4$
 $x = 2, y = 1$

6 $x = 7, y = 2$
 $x = -1, y = -6$

7 $x = 0, y = 5$
 $x = -5, y = 0$

8 $x = \frac{8}{3}, y = -\frac{19}{3}$
 $x = 3, y = 5$

9 $x = -2, y = -4$
 $x = 2, y = 4$

10 $x = \frac{5}{2}, y = 6$
 $x = 3, y = 5$

11 $x = \frac{1+\sqrt{5}}{2}, y = \frac{-1+\sqrt{5}}{2}$
 $x = \frac{1-\sqrt{5}}{2}, y = \frac{-1-\sqrt{5}}{2}$

12 $x = \frac{-1+\sqrt{7}}{2}, y = \frac{3+\sqrt{7}}{2}$
 $x = \frac{-1-\sqrt{7}}{2}, y = \frac{3-\sqrt{7}}{2}$

Solving simultaneous equations graphically

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- You can solve any pair of simultaneous equations by drawing the graph of both equations and finding the point/points of intersection.

Examples

Example 1 Solve the simultaneous equations $y = 5x + 2$ and $x + y = 5$ graphically.

$y = 5 - x$ $y = 5 - x$ has gradient -1 and y-intercept 5. $y = 5x + 2$ has gradient 5 and y-intercept 2. Lines intersect at $x = 0.5, y = 4.5$ Check: First equation $y = 5x + 2$: $4.5 = 5 \times 0.5 + 2$ YES Second equation $x + y = 5$: $0.5 + 4.5 = 5$ YES	1 Rearrange the equation $x + y = 5$ to make y the subject. 2 Plot both graphs on the same grid using the gradients and y-intercepts. 3 The solutions of the simultaneous equations are the point of intersection. 4 Check your solutions by substituting the values into both equations.
--	--

Example 2 Solve the simultaneous equations $y = x - 4$ and $y = x^2 - 4x + 2$ graphically.

<table border="1"> <tr> <td>x</td> <td>0</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>y</td> <td>2</td> <td>-1</td> <td>-2</td> <td>-1</td> <td>2</td> </tr> </table> The line and curve intersect at $x = 3, y = -1$ and $x = 2, y = -2$ Check: First equation $y = x - 4$: $-1 = 3 - 4$ YES $-2 = 2 - 4$ YES Second equation $y = x^2 - 4x + 2$: $-1 = 3^2 - 4 \times 3 + 2$ YES $-2 = 2^2 - 4 \times 2 + 2$ YES	x	0	1	2	3	4	y	2	-1	-2	-1	2	1 Construct a table of values and calculate the points for the quadratic equation. 2 Plot the graph. 3 Plot the linear graph on the same grid using the gradient and y-intercept. $y = x - 4$ has gradient 1 and y-intercept -4. 4 The solutions of the simultaneous equations are the points of intersection. 5 Check your solutions by substituting the values into both equations.
x	0	1	2	3	4								
y	2	-1	-2	-1	2								

Practice

1 Solve these pairs of simultaneous equations graphically.

- $y = 3x - 1$ and $y = x + 3$
- $y = x - 5$ and $y = 7 - 5x$
- $y = 3x + 4$ and $y = 2 - x$

2 Solve these pairs of simultaneous equations graphically.

- $x + y = 0$ and $y = 2x + 6$
- $4x + 2y = 3$ and $y = 3x - 1$
- $2x + y + 4 = 0$ and $2y = 3x - 1$

Hint

Rearrange the equation to make y the subject.

3 Solve these pairs of simultaneous equations graphically.

- a $y = x - 1$ and $y = x^2 - 4x + 3$
- b $y = 1 - 3x$ and $y = x^2 - 3x - 3$
- c $y = 3 - x$ and $y = x^2 + 2x + 5$

4 Solve the simultaneous equations $x + y = 1$ and $x^2 + y^2 = 25$ graphically.

Extend

5 a Solve the simultaneous equations $2x + y = 3$ and $x^2 + y = 4$

- i graphically
- ii algebraically to 2 decimal places.

b Which method gives the more accurate solutions? Explain your answer.

Answers

- 1 a $x = 2, y = 5$
- b $x = 2, y = -3$
- c $x = -0.5, y = 2.5$

- 2 a $x = -2, y = 2$
- b $x = 0.5, y = 0.5$
- c $x = -1, y = -2$

- 3 a $x = 1, y = 0$ and $x = 4, y = 3$
- b $x = -2, y = 7$ and $x = 2, y = -5$
- c $x = -2, y = 5$ and $x = -1, y = 4$

- 4 $x = -3, y = 4$ and $x = 4, y = -3$

- 5 a i $x = 2.5, y = -2$ and $x = -0.5, y = 4$

- ii $x = 2.41, y = -1.83$ and $x = -0.41, y = 3.83$

b Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.

Linear inequalities

AL LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. $<$ becomes $>$.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq x < 4$	Divide all three terms by 4.
--------------------------------------	------------------------------

Example 2 Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.
--	------------------------------

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	1 Add 5 to both sides. 2 Divide both sides by 2.
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Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	1 Subtract 2 from both sides. 2 Divide both sides by -5 . Remember to reverse the inequality when dividing by a negative number.
--	--

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$ $x > 5$	1 Expand the brackets. 2 Add $3x$ to both sides. 3 Add 8 to both sides. 4 Divide both sides by 7.
--	--

Practice

1 Solve these inequalities.

- a $4x > 16$ b $5x - 7 \leq 3$ c $1 \geq 3x + 4$
d $5 - 2x < 12$ e $\frac{x}{2} \geq 5$ f $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

- a $\frac{x}{5} < -4$ b $10 \geq 2x + 3$ c $7 - 3x > -5$

3 Solve

- a $2 - 4x \geq 18$ b $3 \leq 7x + 10 < 45$ c $6 - 2x \geq 4$
d $4x + 17 < 2 - x$ e $4 - 5x < -3x$ f $-4x \geq 24$

4 Solve these inequalities.

- a $3t + 1 < t + 6$ b $2(3n - 1) \geq n + 5$

5 Solve.

- a $3(2 - x) > 2(4 - x) + 4$ b $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Answers

- | | | | | | | |
|---|---|--------------------|---|----------------------|---|-------------|
| 1 | a | $x > 4$ | b | $x \leq 2$ | c | $x \leq -1$ |
| | d | $x > -\frac{7}{2}$ | e | $x \geq 10$ | f | $x < -15$ |
| 2 | a | $x < -20$ | b | $x \leq 3.5$ | c | $x < 4$ |
| 3 | a | $x \leq -4$ | b | $-1 \leq x < 5$ | c | $x \leq 1$ |
| | d | $x < -3$ | e | $x > 2$ | f | $x \leq -6$ |
| 4 | a | $t < \frac{5}{2}$ | b | $n \geq \frac{7}{5}$ | | |
| 5 | a | $x < -6$ | b | $x < \frac{3}{2}$ | | |
| 6 | $x > 5$ (which also satisfies $x > 3$) | | | | | |

Quadratic inequalities

A LEVEL LINKS

Scheme of work: 1d. Inequalities — linear and quadratic (including graphical solutions)

Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$	It is above the x-axis where $x^2 + 5x + 6 > 0$ This part of the graph is not needed as this is where $x^2 + 5x + 6 < 0$ $x < -3$ or $x > -2$	1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = (x + 3)(x + 2)$ 3 Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$ 4 Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$
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Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$	$0 \leq x \leq 5$	1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = x(x - 5)$ 3 Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$ 4 Write down the values which satisfy the inequality $x^2 - 5x \leq 0$
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Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$	$-5 \leq x \leq 2$	1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$ 3 Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$ 3 Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$
--	--------------------------------------	---

Practice

- 1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$
- 2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$
- 3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$
- 4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$
- 5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

- 6 $x^2 + x \leq 6$
- 7 $x(2x - 9) < -10$
- 8 $6x^2 \geq 15 + x$

Answers

1 $-7 \leq x \leq 4$

2 $x \leq -2$ or $x \geq 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \leq x \leq 4$

6 $-3 \leq x \leq 2$

7 $2 < x < 2\frac{1}{2}$

8 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

Quadratic inequalities

A LEVEL LINKS

Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- First replace the inequality sign by = and solve the quadratic equation.
- Sketch the graph of the quadratic function.
- Use the graph to find the values which satisfy the quadratic inequality.

Examples

Example 1 Find the set of values of x which satisfy $x^2 + 5x + 6 > 0$

$x^2 + 5x + 6 = 0$ $(x + 3)(x + 2) = 0$ $x = -3$ or $x = -2$		1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = (x + 3)(x + 2)$ 3 Identify on the graph where $x^2 + 5x + 6 > 0$, i.e. where $y > 0$ 4 Write down the values which satisfy the inequality $x^2 + 5x + 6 > 0$
--	--	---

Example 2 Find the set of values of x which satisfy $x^2 - 5x \leq 0$

$x^2 - 5x = 0$ $x(x - 5) = 0$ $x = 0$ or $x = 5$		1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = x(x - 5)$ 3 Identify on the graph where $x^2 - 5x \leq 0$, i.e. where $y \leq 0$ 4 Write down the values which satisfy the inequality $x^2 - 5x \leq 0$
--	--	---

Example 3 Find the set of values of x which satisfy $-x^2 - 3x + 10 \geq 0$

$-x^2 - 3x + 10 = 0$ $(-x + 2)(x + 5) = 0$ $x = 2$ or $x = -5$		1 Solve the quadratic equation by factorising. 2 Sketch the graph of $y = (-x + 2)(x + 5) = 0$ 3 Identify on the graph where $-x^2 - 3x + 10 \geq 0$, i.e. where $y \geq 0$ 3 Write down the values which satisfy the inequality $-x^2 - 3x + 10 \geq 0$
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Practice

- 1 Find the set of values of x for which $(x + 7)(x - 4) \leq 0$
- 2 Find the set of values of x for which $x^2 - 4x - 12 \geq 0$
- 3 Find the set of values of x for which $2x^2 - 7x + 3 < 0$
- 4 Find the set of values of x for which $4x^2 + 4x - 3 > 0$
- 5 Find the set of values of x for which $12 + x - x^2 \geq 0$

Extend

Find the set of values which satisfy the following inequalities.

- 6 $x^2 + x \leq 6$
- 7 $x(2x - 9) < -10$
- 8 $6x^2 \geq 15 + x$

Answers

1 $-7 \leq x \leq 4$

2 $x \leq -2$ or $x \geq 6$

3 $\frac{1}{2} < x < 3$

4 $x < -\frac{3}{2}$ or $x > \frac{1}{2}$

5 $-3 \leq x \leq 4$

6 $-3 \leq x \leq 2$

7 $2 < x < 2\frac{1}{2}$

8 $x \leq -\frac{3}{2}$ or $x \geq \frac{5}{3}$

Translating graphs

A LEVEL LINKS

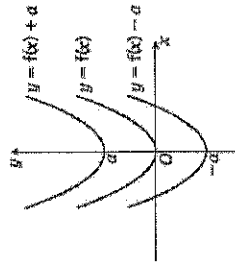
Scheme of work: 1f. Transformations – transforming graphs – $f(x)$ notation

Key points

- The transformation $y = f(x) \pm a$ is a translation of $y = f(x)$ parallel to the y -axis; it is a vertical translation.

As shown on the graph,

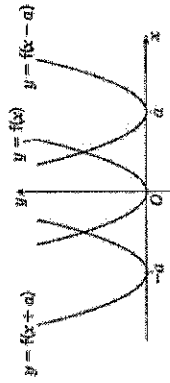
- $y = f(x) + a$ translates $y = f(x)$ up
- $y = f(x) - a$ translates $y = f(x)$ down.



- The transformation $y = f(x \pm a)$ is a translation of $y = f(x)$ parallel to the x -axis; it is a horizontal translation.

As shown on the graph,

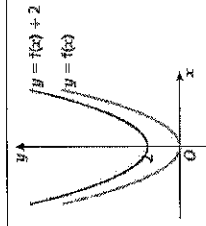
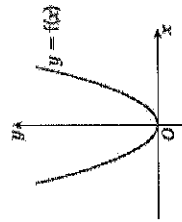
- $y = f(x + a)$ translates $y = f(x)$ to the left
- $y = f(x - a)$ translates $y = f(x)$ to the right.



Examples

Example 1 The graph shows the function $y = f(x)$.

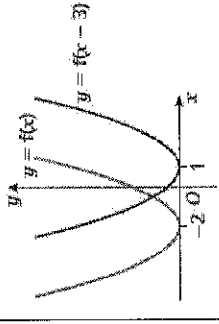
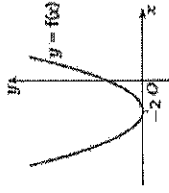
Sketch the graph of $y = f(x) + 2$.



For the function $y = f(x) + 2$ translate the function $y = f(x)$ 2 units up.

Example 2 The graph shows the function $y = f(x)$.

Sketch the graph of $y = f(x - 3)$.

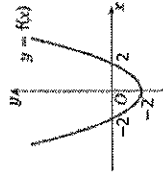


For the function $y = f(x - 3)$ translate the function $y = f(x)$ 3 units right.

Practice

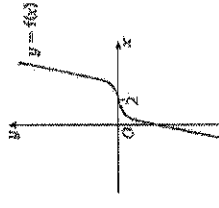
1 The graph shows the function $y = f(x)$.

Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.



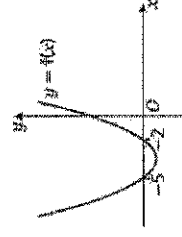
2 The graph shows the function $y = f(x)$.

Copy the graph and on the same axes sketch and label the graphs of $y = f(x + 3)$ and $y = f(x) - 3$.

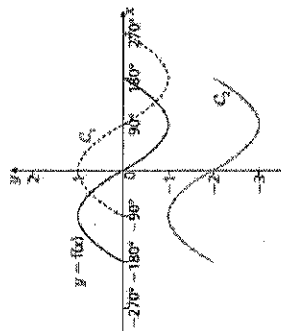


3 The graph shows the function $y = f(x)$.

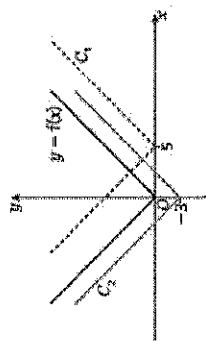
Copy the graph and on the same axes sketch the graph of $y = f(x - 5)$.



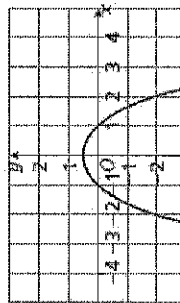
- 4 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2 . Write down the equations of the translated curves C_1 and C_2 in function form.



- 5 The graph shows the function $y = f(x)$ and two transformations of $y = f(x)$, labelled C_1 and C_2 . Write down the equations of the translated curves C_1 and C_2 in function form.



- 6 The graph shows the function $y = f(x)$.
- Sketch the graph of $y = f(x) + 2$
 - Sketch the graph of $y = f(x + 2)$



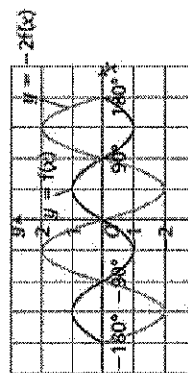
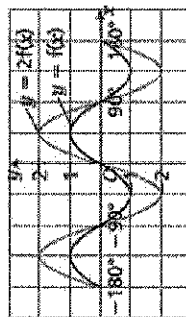
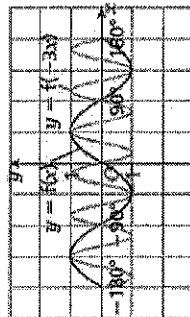
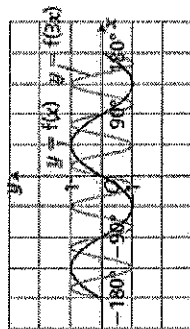
Stretching graphs

A LEVEL LINKS

Scheme of work: 1f Transformations – transforming graphs – $f(x)$ notation
Textbook: Pure Year 1, 4.6 Stretching graphs

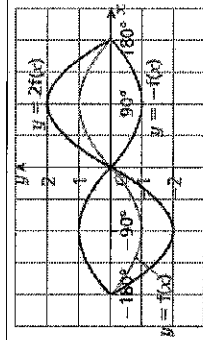
Key points

- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x -axis.
- The transformation $y = f(-ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel to the x -axis and then a reflection in the y -axis.
- The transformation $y = af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y -axis.
- The transformation $y = -af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel to the y -axis and then a reflection in the x -axis.



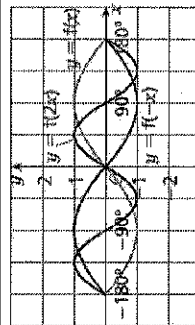
Examples

Example 3 The graph shows the function $y = f(x)$. Sketch and label the graphs of $y = 2f(x)$ and $y = -f(x)$.



The function $y = 2f(x)$ is a vertical stretch of $y = f(x)$ with scale factor 2 parallel to the y -axis.
The function $y = -f(x)$ is a reflection of $y = f(x)$ in the x -axis.

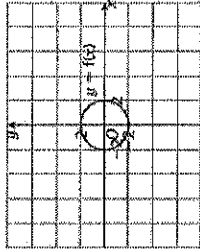
Example 4 The graph shows the function $y = f(x)$. Sketch and label the graphs of $y = f(2x)$ and $y = f(-x)$.



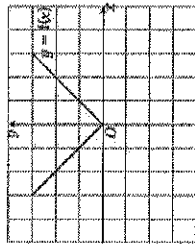
The function $y = f(2x)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{2}$ parallel to the x -axis.
The function $y = f(-x)$ is a reflection of $y = f(x)$ in the y -axis.

Practice

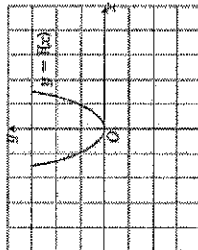
7 The graph shows the function $y = f(x)$.
a Copy the graph and on the same axes sketch and label the graph of $y = 3f(x)$.
b Make another copy of the graph and on the same axes sketch and label the graph of $y = f(2x)$.



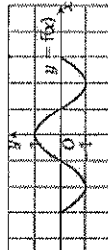
8 The graph shows the function $y = f(x)$. Copy the graph and on the same axes sketch and label the graphs of $y = -2f(x)$ and $y = f(3x)$.



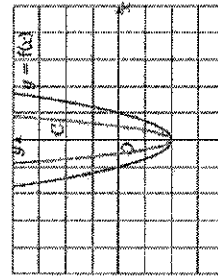
9 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch and label the graphs of $y = -f(x)$ and $y = f(\frac{1}{2}x)$.



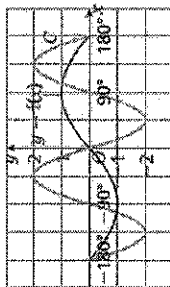
10 The graph shows the function $y = f(x)$. Copy the graph and, on the same axes, sketch the graph of $y = -f(2x)$.



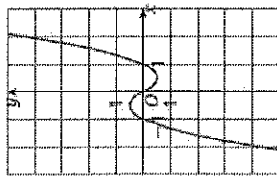
11 The graph shows the function $y = f(x)$ and a transformation, labelled C . Write down the equation of the translated curve C in function form.



- 12 The graph shows the function $y = f(x)$ and a transformation labelled C. Write down the equation of the translated curve C in function form.



- 13 The graph shows the function $y = f(x)$.
 a Sketch the graph of $y = -f(x)$.
 b Sketch the graph of $y = 2f(x)$.

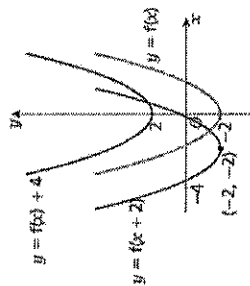


Extend

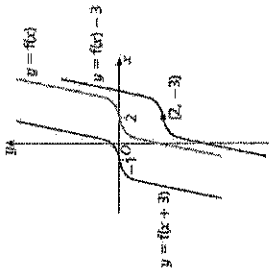
- 14 a Sketch and label the graph of $y = f(x)$, where $f(x) = (x-1)(x+1)$.
 b On the same axes, sketch and label the graphs of $y = f(x) - 2$ and $y = f(x) + 2$.
- 15 a Sketch and label the graph of $y = f(x)$, where $f(x) = -(x+1)(x-2)$.
 b On the same axes, sketch and label the graph of $y = f(-\frac{1}{2}x)$.

Answers

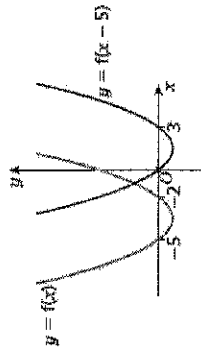
1



2



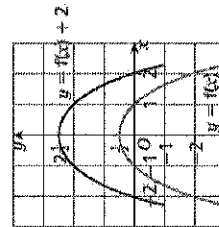
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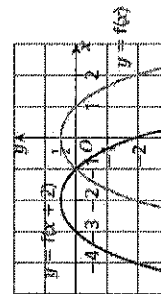
- 4 $C_1: y = f(x - 90^\circ)$
 $C_2: y = f(x) - 2$

- 5 $C_1: y = f(x - 5)$
 $C_2: y = f(x) - 3$

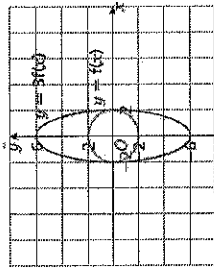
6 a



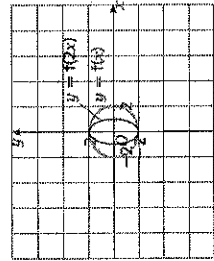
b



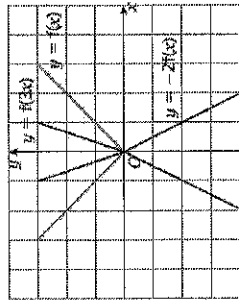
7 a



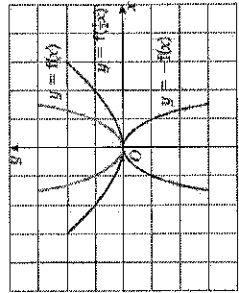
b



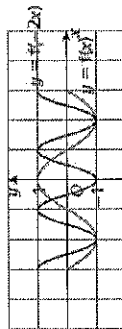
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9



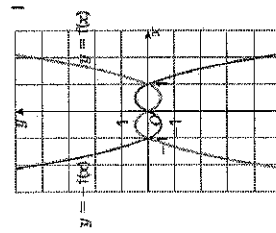
10



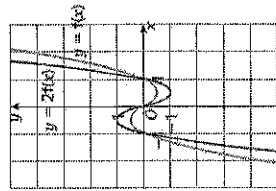
11 $y = f(2x)$

12 $y = -2f(2x)$ or $y = 2f(-2x)$

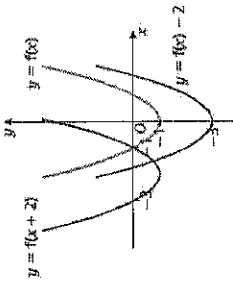
13 a



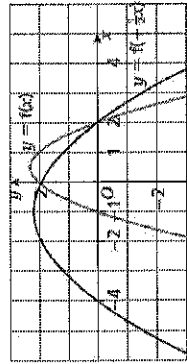
b



14



15



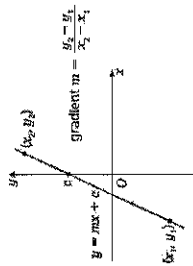
Straight line graphs

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y -intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a , b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y -intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$$m = -\frac{1}{2} \text{ and } c = 3$$

$$\text{So } y = -\frac{1}{2}x + 3$$

$$\frac{1}{2}x + y - 3 = 0$$

$$x + 2y - 6 = 0$$

- A straight line has equation $y = mx + c$. Substitute the gradient and y -intercept given in the question into this equation.
- Rearrange the equation so all the terms are on one side and 0 is on the other side.
- Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y -intercept of the line with the equation $3y - 2x + 4 = 0$.

$$3y - 2x + 4 = 0$$

$$3y = 2x - 4$$

$$y = \frac{2}{3}x - \frac{4}{3}$$

$$\text{Gradient} = m = \frac{2}{3}$$

$$y\text{-intercept} = c = -\frac{4}{3}$$

- Make y the subject of the equation.
- Divide all the terms by three to get the equation in the form $y = \dots$
- In the form $y = mx + c$, the gradient is m and the y -intercept is c .

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$	1 Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	2 Substitute the coordinates $x = 5$ and $y = 13$ into the equation.
$13 = 15 + c$ $c = -2$	3 Simplify and solve the equation.
$y = 3x - 2$	4 Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2, x_2 = 8, y_1 = 4 \text{ and } y_2 = 7$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{8 - 2} = \frac{3}{6} = \frac{1}{2}$	1 Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \frac{1}{2}x + c$	2 Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \frac{1}{2} \times 2 + c$	3 Substitute the coordinates of either point into the equation.
$c = 3$	4 Simplify and solve the equation.
$y = \frac{1}{2}x + 3$	5 Substitute $c = 3$ into the equation $y = \frac{1}{2}x + c$

Practice

1 Find the gradient and the y -intercept of the following equations.

a $y = 3x + 5$ b $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ d $x + y = 5$

e $2x - 3y - 7 = 0$ f $5x + y - 4 = 0$

Hint
Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	y -intercept	Equation of the line
5	0	
-3	2	
4	-7	

- 3 Find, in the form $ax + by + c = 0$ where a , b and c are integers, an equation for each of the lines with the following gradients and y -intercepts.

- a** gradient $-\frac{1}{2}$, y -intercept -7 **b** gradient 2 , y -intercept 0
c gradient $\frac{2}{3}$, y -intercept 4 **d** gradient -1.2 , y -intercept -2

- 4 Write an equation for the line which passes through the point $(2, 5)$ and has gradient 4 .

- 5 Write an equation for the line which passes through the point $(6, 3)$ and has gradient $-\frac{2}{3}$

- 6 Write an equation for the line passing through each of the following pairs of points.

- a** $(4, 5)$, $(10, 17)$ **b** $(0, 6)$, $(-4, 8)$
c $(-1, -7)$, $(5, 23)$ **d** $(3, 10)$, $(4, 7)$

Extend

- 7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Answers

- 1 a** $m = 3$, $c = 5$ **b** $m = -\frac{1}{2}$, $c = -7$
c $m = 2$, $c = -\frac{3}{2}$ **d** $m = -1$, $c = 5$
e $m = \frac{2}{3}$, $c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5$, $c = 4$

2

Gradient	y -intercept	Equation of the line
5	0	$y = 5x$
-3	2	$y = -3x + 2$
4	-7	$y = 4x - 7$

- 3 a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

- c** $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

- 4** $y = 4x - 3$

- 5** $y = -\frac{2}{3}x + 7$

- 6 a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

- c** $y = 5x - 2$ **d** $y = -3x + 19$

- 7** $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the y -intercept is 3.

The line intercepts the axes at $(0, 3)$ and $(2, 0)$.

Students may sketch the line or give coordinates that lie on the line such as $(1, \frac{3}{2})$ or $(4, -3)$.

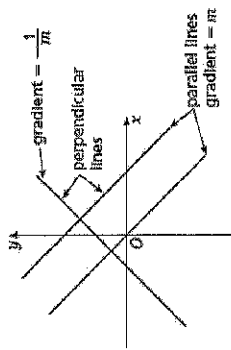
Parallel and perpendicular lines

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point $(4, 9)$.

$y = 2x + 4$ $m = 2$ $y = 2x + c$ $9 = 2 \times 4 + c$ $9 = 8 + c$ $c = 1$ $y = 2x + 1$	- As the lines are parallel they have the same gradient. - Substitute $m = 2$ into the equation of a straight line $y = mx + c$. - Substitute the coordinates into the equation $y = 2x + c$ - Simplify and solve the equation. - Substitute $c = 1$ into the equation $y = 2x + c$
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Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point $(-2, 5)$.

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$ $5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	- As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. - Substitute $m = -\frac{1}{2}$ into $y = mx + c$. - Substitute the coordinates $(-2, 5)$ into the equation $y = -\frac{1}{2}x + c$ - Simplify and solve the equation. - Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.
--	--

Example 3 A line passes through the points $(0, 5)$ and $(9, -1)$. Find the equation of the line which is perpendicular to the line and passes through its midpoint

$x_1 = 0, x_2 = 9, y_1 = 5$ and $y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 5}{9 - 0} = \frac{-6}{9} = -\frac{2}{3}$ $-\frac{1}{m} = -\frac{1}{-\frac{2}{3}} = \frac{3}{2}$ $y = \frac{3}{2}x + c$ Midpoint $= \left(\frac{0+9}{2}, \frac{5+(-1)}{2} \right) = \left(\frac{9}{2}, 2 \right)$ $2 = \frac{3}{2} \times \frac{9}{2} + c$ $c = -\frac{19}{4}$ $y = \frac{3}{2}x - \frac{19}{4}$	- Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line. - As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. - Substitute the gradient into the equation $y = mx + c$. - Work out the coordinates of the midpoint of the line. - Substitute the coordinates of the midpoint into the equation. - Simplify and solve the equation. - Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.
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Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

- a $y = 3x + 1$ (3, 2) b $y = 3 - 2x$ (1, 3)
c $2x + 4y + 3 = 0$ (6, -3) d $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point $(-5, 3)$.

Hint
If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

- a $y = 2x - 6$ (4, 0) b $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)
c $x - 4y - 4 = 0$ (5, 15) d $5y + 2x - 5 = 0$ (6, 7)

- 4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a $(4, 3), (-2, -9)$ b $(0, 3), (-10, 8)$

Extend

- 5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$ b $y = 3x$ c $y = 4x - 3$
 $y = 2x - 7$ $2x + y - 3 = 0$ $4y + x = 2$

d $3x - y + 5 = 0$ e $2x + 5y - 1 = 0$ f $2x - y = 6$
 $x + 3y = 1$ $y = 2x + 7$ $6x - 3y + 3 = 0$

- 6 The straight line L_1 passes through the points A and B with coordinates $(-4, 4)$ and $(2, 1)$, respectively.

a Find the equation of L_1 in the form $ax + by + c = 0$

The line L_2 is parallel to the line L_1 and passes through the point C with coordinates $(-8, 3)$.

b Find the equation of L_2 in the form $ax + by + c = 0$

The line L_3 is perpendicular to the line L_1 and passes through the origin.

c Find an equation of L_3

Answers

1 a $y = 3x - 7$ b $y = -2x + 5$
c $y = -\frac{1}{2}x$ d $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 a $y = -\frac{1}{2}x + 2$ b $y = 3x + 7$

c $y = -4x + 35$ d $y = \frac{5}{2}x - 8$

4 a $y = -\frac{1}{2}x$ b $y = 2x$

5 a Parallel b Neither c Perpendicular
d Perpendicular e Neither f Parallel

6 a $x + 2y - 4 = 0$ b $x + 2y + 2 = 0$ c $y = 2x$

Pythagoras' theorem

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- In a right-angled triangle the longest side is called the hypotenuse.
- Pythagoras' theorem states that for a right-angled triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

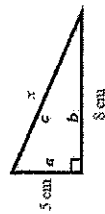
$$c^2 = a^2 + b^2$$



Examples

Example 1 Calculate the length of the hypotenuse.
Give your answer to 3 significant figures.

$$c^2 = a^2 + b^2$$



$$\begin{aligned} x^2 &= 5^2 + 8^2 \\ x^2 &= 25 + 64 \\ x^2 &= 89 \\ x &= \sqrt{89} \end{aligned}$$

$$\begin{aligned} x &= 9.433\ 981\ 13... \\ x &= 9.43\text{ cm} \end{aligned}$$

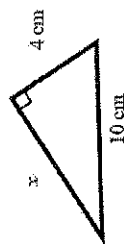
1 Always start by stating the formula for Pythagoras' theorem and labelling the hypotenuse c and the other two sides a and b .

2 Substitute the values of a , b and c into the formula for Pythagoras' theorem.

3 Use a calculator to find the square root.

4 Round your answer to 3 significant figures and write the units with your answer.

Example 2 Calculate the length x .
Give your answer in surd form.



$$c^2 = a^2 + b^2$$

$$10^2 = x^2 + 4^2$$

$$100 = x^2 + 16$$

$$x^2 = 84$$

$$x = \sqrt{84}$$

$$x = 2\sqrt{21}\text{ cm}$$

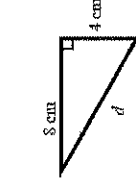
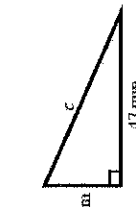
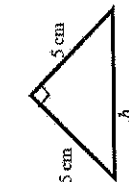
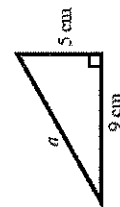
1 Always start by stating the formula for Pythagoras' theorem.

2 Substitute the values of a , b and c into the formula for Pythagoras' theorem.

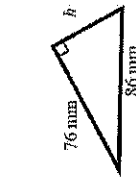
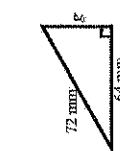
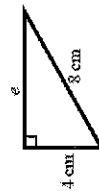
3 Simplify the surd where possible and write the units in your answer.

Practice

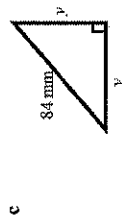
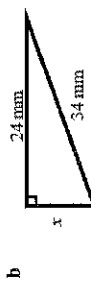
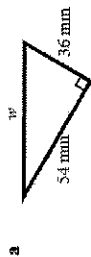
1 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.



2 Work out the length of the unknown side in each triangle.
Give your answers in surd form.



- 3 Work out the length of the unknown side in each triangle. Give your answers in surd form.



- 4 A rectangle has length 84 mm and width 45 mm. Calculate the length of the diagonal of the rectangle. Give your answer correct to 3 significant figures.

Hint

Draw a sketch of the rectangle.

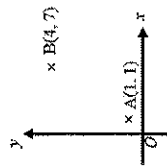
Extend

- 5 A yacht is 40 km due North of a lighthouse. A rescue boat is 50 km due East of the same lighthouse. Work out the distance between the yacht and the rescue boat. Give your answer correct to 3 significant figures.

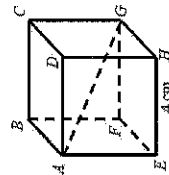
Hint

Draw a diagram using the information given in the question.

- 6 Points A and B are shown on the diagram. Work out the length of the line AB. Give your answer in surd form.



- 7 A cube has length 4 cm. Work out the length of the diagonal AG. Give your answer in surd form.



Answers

- 1 a 10.3 cm b 7.07 cm
c 58.6 mm d 8.94 cm
- 2 a $4\sqrt{3}$ cm b $2\sqrt{21}$ cm
c $8\sqrt{17}$ mm d $18\sqrt{5}$ mm
- 3 a $18\sqrt{13}$ mm b $2\sqrt{145}$ mm
c $42\sqrt{2}$ mm d $6\sqrt{89}$ mm
- 4 95.3 mm
- 5 64.0 km
- 6 $3\sqrt{5}$ units
- 7 $4\sqrt{3}$ cm

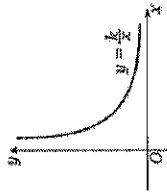
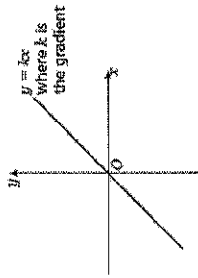
Proportion

A LEVEL LINKS

Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- Two quantities are in direct proportion when, as one quantity increases, the other increases at the same rate. Their ratio remains the same.
- 'y is directly proportional to x' is written as $y \propto x$. If $y \propto x$ then $y = kx$, where k is a constant.
- When x is directly proportional to y, the graph is a straight line passing through the origin.
- Two quantities are in inverse proportion when, as one quantity increases, the other decreases at the same rate.
- 'y is inversely proportional to x' is written as $y \propto \frac{1}{x}$. If $y \propto \frac{1}{x}$ then $y = \frac{k}{x}$, where k is a constant.
- When x is inversely proportional to y the graph is the same shape as the graph of $y = \frac{1}{x}$

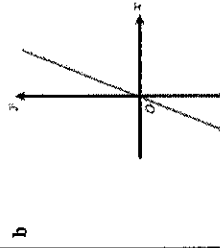


Examples

Example 1 y is directly proportional to x.

- When $y = 16$, $x = 5$.
- Find x when $y = 30$.
 - Sketch the graph of the formula.

a $y \propto x$ $y = kx$ $16 = k \times 5$ $k = 3.2$ $y = 3.2x$ When $y = 30$, $30 = 3.2 \times x$ $x = 9.375$	- Write y is directly proportional to x, using the symbol $\propto$. - Write the equation using k. - Substitute $y = 16$ and $x = 5$ into $y = kx$. - Solve the equation to find k. - Substitute the value of k back into the equation $y = kx$. - Substitute $y = 30$ into $y = 3.2x$ and solve to find x when $y = 30$.
---	--



- The graph of $y = 3.2x$ is a straight line passing through (0, 0) with a gradient of 3.2.

Example 2

y is directly proportional to x^2 .

- When $x = 3$, $y = 45$.
- Find y when $x = 5$.
 - Find x when $y = 20$.

- a** $y \propto x^2$
- $y = kx^2$
- $45 = k \times 3^2$
- $k = 5$
- $y = 5x^2$
- When $x = 5$,
- $y = 5 \times 5^2$
- $y = 125$
- b** $20 = 5 \times x^2$
- $x^2 = 4$
- $x = \pm 2$

- Write y is directly proportional to x^2 , using the symbol $\propto$.
- Write the equation using k.
- Substitute $y = 45$ and $x = 3$ into $y = kx^2$.
- Solve the equation to find k.
- Substitute the value of k back into the equation $y = kx^2$.
- Substitute $x = 5$ into $y = 5x^2$ and solve to find y when $x = 5$.
- Substitute $y = 20$ into $y = 5x^2$ and solve to find x when $y = 4$.

Example 3

P is inversely proportional to Q.

- When $P = 100$, $Q = 10$.
- Find Q when $P = 20$.

- $P \propto \frac{1}{Q}$
- $P = \frac{k}{Q}$
- $100 = \frac{k}{10}$
- $k = 1000$
- $P = \frac{1000}{Q}$
- $20 = \frac{1000}{Q}$
- $Q = \frac{1000}{20} = 50$

- Write P is inversely proportional to Q, using the symbol $\propto$.
- Write the equation using k.
- Substitute $P = 100$ and $Q = 10$.
- Solve the equation to find k.
- Substitute the value of k into $P = \frac{k}{Q}$.
- Substitute $P = 20$ into $P = \frac{1000}{Q}$ and solve to find Q when $P = 20$.

Practice

- 1 Paul gets paid an hourly rate. The amount of pay (£ P) is directly proportional to the number of hours (h) he works.
When he works 8 hours he is paid £56.
If Paul works for 11 hours, how much is he paid?
- 2 x is directly proportional to y .
 $x = 35$ when $y = 5$.
 - a Find a formula for x in terms of y .
 - b Sketch the graph of the formula.
 - c Find x when $y = 13$.
 - d Find y when $x = 63$.
- 3 Q is directly proportional to the square of Z .
 $Q = 48$ when $Z = 4$.
 - a Find a formula for Q in terms of Z .
 - b Sketch the graph of the formula.
 - c Find Q when $Z = 5$.
 - d Find Z when $Q = 300$.
- 4 y is directly proportional to the square of x .
 $x = 2$ when $y = 10$.
 - a Find a formula for y in terms of x .
 - b Sketch the graph of the formula.
 - c Find x when $y = 90$.
- 5 B is directly proportional to the square root of C .
 $C = 25$ when $B = 10$.
 - a Find B when $C = 64$.
 - b Find C when $B = 20$.
- 6 C is directly proportional to D .
 $C = 100$ when $D = 150$.
Find C when $D = 450$.
- 7 y is directly proportional to x .
 $x = 27$ when $y = 9$.
Find x when $y = 3.7$.
- 8 m is proportional to the cube of n .
 $m = 54$ when $n = 3$.
Find n when $m = 250$.

Hint

Substitute the values given for P and h into the formula to calculate k .

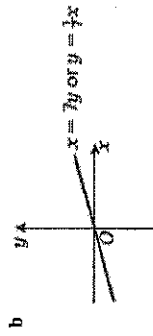
Extend

- 9 s is inversely proportional to t .
 - a Given that $s = 2$ when $t = 2$, find a formula for s in terms of t .
 - b Sketch the graph of the formula.
 - c Find t when $s = 1$.
- 10 a is inversely proportional to b .
 $a = 5$ when $b = 20$.
 - a Find a when $b = 50$.
 - b Find b when $a = 10$.
- 11 v is inversely proportional to w .
 $w = 4$ when $v = 20$.
 - a Find a formula for v in terms of w .
 - b Sketch the graph of the formula.
 - c Find w when $v = 2$.
- 12 L is inversely proportional to W .
 $L = 12$ when $W = 3$.
Find W when $L = 6$.
- 13 s is inversely proportional to t .
 $s = 6$ when $t = 12$.
 - a Find s when $t = 3$.
 - b Find t when $s = 18$.
- 14 y is inversely proportional to x^2 .
 $y = 4$ when $x = 2$.
Find y when $x = 4$.
- 15 y is inversely proportional to the square root of x .
 $x = 25$ when $y = 1$.
Find x when $y = 5$.
- 16 a is inversely proportional to b .
 $a = 0.05$ when $b = 4$.
 - a Find a when $b = 2$.
 - b Find b when $a = 2$.

Answers

1 E77

2 a $x = 7y$

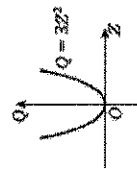


b

c 91

d 9

3 a $Q = 3Z^2$

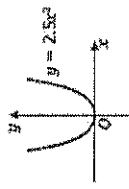


b

c 75

d ± 10

4 a $y = 2.5x^2$



b

c ± 6

5 a 16

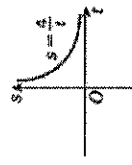
b 100

6 300

7 11.1

8 5

9 a $s = \frac{4}{t}$



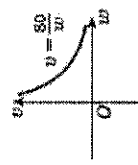
b

c 4

10 a 2

b 10

11 a $v = \frac{80}{w}$



b

c 40

12 6

13 a 24

b 4

14 1

15 1

16 a 0.1

b 0.1

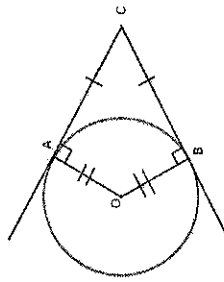
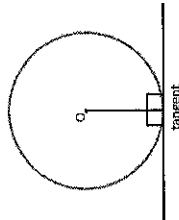
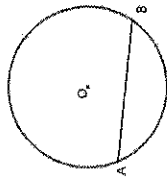
Circle theorems

A LEVEL LINKS

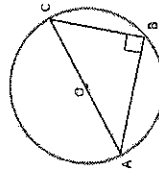
Scheme of work: 2b. Circles – equation of a circle, geometric problems on a grid

Key points

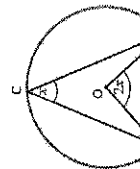
- A chord is a straight line joining two points on the circumference of a circle.
So AB is a chord.
- A tangent is a straight line that touches the circumference of a circle at only one point.
The angle between a tangent and the radius is 90° .



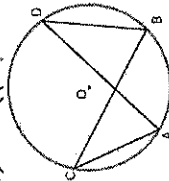
- Two tangents on a circle that meet at a point outside the circle are equal in length.
So $AC = BC$.



- The angle in a semicircle is a right angle.
So angle $ABC = 90^\circ$.



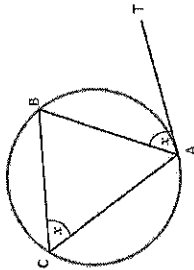
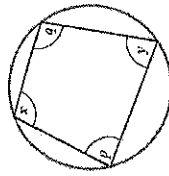
- When two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.
So angle $AOB = 2 \times \text{angle } ACB$.



- Angles subtended by the same arc at the circumference are equal. This means that angles in the same segment are equal.

So angle $ACB = \text{angle } ADB$ and
angle $CAD = \text{angle } CBD$.

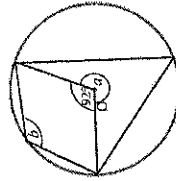
- A cyclic quadrilateral is a quadrilateral with all four vertices on the circumference of a circle.
Opposite angles in a cyclic quadrilateral total 180° .
So $x + y = 180^\circ$ and $p + q = 180^\circ$.
- The angle between a tangent and chord is equal to the angle in the alternate segment, this is known as the alternate segment theorem.
So angle $BAT = \text{angle } ACB$.



Examples

Example 1

Work out the size of each angle marked with a letter.
Give reasons for your answers.



Angle $a = 360^\circ - 92^\circ$
 $= 268^\circ$
as the angles in a full turn total 360° .

Angle $b = 268^\circ \div 2$
 $= 134^\circ$

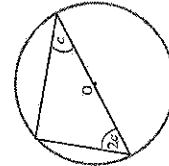
as when two angles are subtended by the same arc, the angle at the centre of a circle is twice the angle at the circumference.

1 The angles in a full turn total 360° .

2 Angles a and b are subtended by the same arc, so angle b is half of angle a .

Example 2

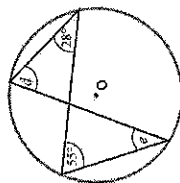
Work out the size of the angles in the triangle.
Give reasons for your answers.



Angles are 90°, $2c$ and c. $90^\circ + 2c + c = 180^\circ$ $90^\circ + 3c = 180^\circ$ $3c = 90^\circ$ $c = 30^\circ$ $2c = 60^\circ$ The angles are 30°, 60° and 90° as the angle in a semi-circle is a right angle and the angles in a triangle total 180°.	1 The angle in a semi-circle is a right angle. 2 Angles in a triangle total 180°. 3 Simplify and solve the equation.
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Example 3

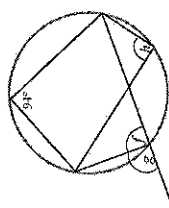
Work out the size of each angle marked with a letter. Give reasons for your answers.



Angle $d = 55^\circ$ as angles subtended by the same arc are equal. Angle $e = 28^\circ$ as angles subtended by the same arc are equal.	1 Angles subtended by the same arc are equal so angle 55° and angle d are equal. 2 Angles subtended by the same arc are equal so angle 28° and angle e are equal.
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Example 4

Work out the size of each angle marked with a letter. Give reasons for your answers.

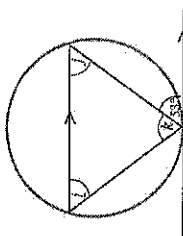


Angle $f = 180^\circ - 94^\circ = 86^\circ$ as opposite angles in a cyclic quadrilateral total 180°.	1 Opposite angles in a cyclic quadrilateral total 180° so angle 94° and angle f total 180°. (continued on next page)
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Angle $g = 180^\circ - 86^\circ = 94^\circ$ as angles on a straight line total 180°. Angle $h = \text{angle } f = 86^\circ$ as angles subtended by the same arc are equal.	2 Angles on a straight line total 180° so angle f and angle g total 180°. 3 Angles subtended by the same arc are equal so angle f and angle h are equal.
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Example 5

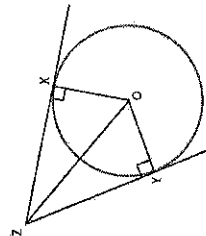
Work out the size of each angle marked with a letter. Give reasons for your answers.



Angle $j = 53^\circ$ because of the alternate segment theorem. Angle $j = 53^\circ$ because it is the alternate angle to 53°. Angle $k = 180^\circ - 53^\circ - 53^\circ = 74^\circ$ as angles in a triangle total 180°.	1 The angle between a tangent and chord is equal to the angle in the alternate segment. 2 As there are two parallel lines, angle 53° is equal to angle j because they are alternate angles. 3 The angles in a triangle total 180°, so $i + j + k = 180^\circ$.
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Example 6

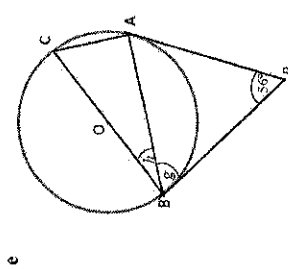
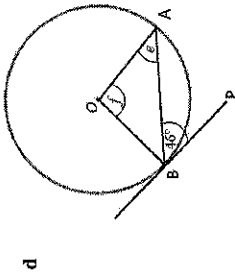
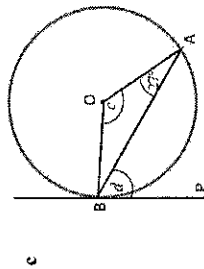
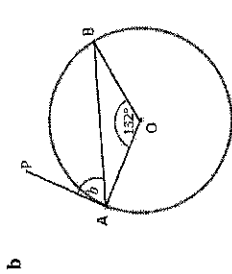
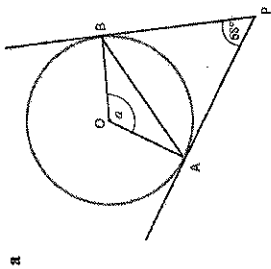
XZ and YZ are two tangents to a circle with centre O . Prove that triangles XZO and YZO are congruent.



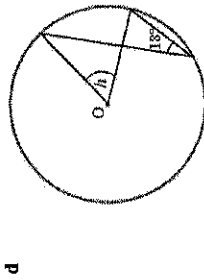
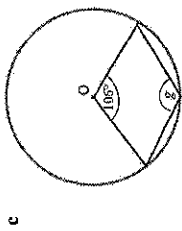
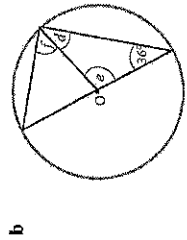
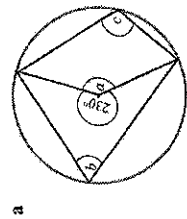
Angle $OXZ = 90^\circ$ and angle $OYZ = 90^\circ$ as the angles in a semicircle are right angles. OZ is a common line and is the hypotenuse in both triangles. $OX = OY$ as they are radii of the same circle. So triangles XZO and YZO are congruent, RHS.	For two triangles to be congruent you need to show one of the following. • All three corresponding sides are equal (SSS). • Two corresponding sides and the included angle are equal (SAS). • One side and two corresponding angles are equal (ASA). • A right angle, hypotenuse and a shorter side are equal (RHS).
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Practice

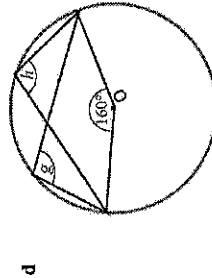
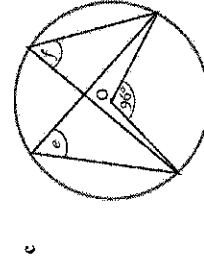
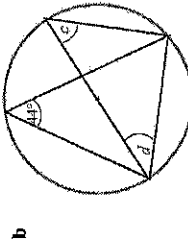
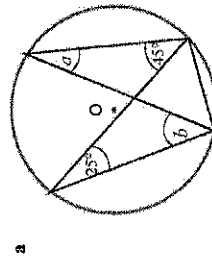
- 1 Work out the size of each angle marked with a letter. Give reasons for your answers.



- 2 Work out the size of each angle marked with a letter. Give reasons for your answers.



- 3 Work out the size of each angle marked with a letter. Give reasons for your answers.



Hint

The reflex angle at point O and angle g are subtended by the same arc. So the reflex angle is twice the size of angle g.

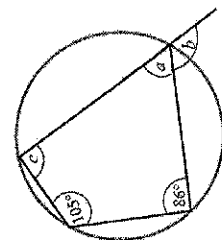
Hint

Angle 18° and angle h are subtended by the same arc.

Hint

One of the angles is in a semicircle.

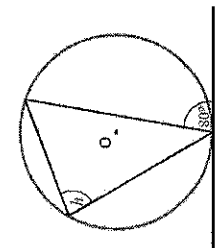
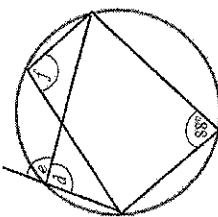
- 4 Work out the size of each angle marked with a letter.
Give reasons for your answers.



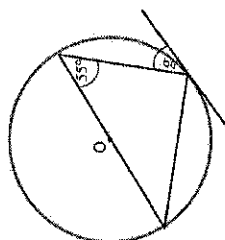
Hint

An exterior angle of a cyclic quadrilateral is equal to the opposite interior angle.

c



d



Hint

One of the angles is in a semicircle.

Extend

- 5 Prove the alternate segment theorem.

Answers

- 1
 - a $a = 112^\circ$, angle $OAP = \text{angle } OBP = 90^\circ$ and angles in a quadrilateral total 360° .
 - b $b = 66^\circ$, triangle OAB is isosceles, Angle $OAP = 90^\circ$ as AP is tangent to the circle.
 - c $c = 126^\circ$, triangle OAB is isosceles.
 $d = 63^\circ$, Angle $OBP = 90^\circ$ as BP is tangent to the circle.
 - d $e = 44^\circ$, the triangle is isosceles, so angles e and angle OBA are equal. The angle $OBP = 90^\circ$ as BP is tangent to the circle.
 $f = 92^\circ$, the triangle is isosceles.
 - e $g = 62^\circ$, triangle ABP is isosceles as AP and BP are both tangents to the circle.
 $h = 28^\circ$, the angle $OBP = 90^\circ$.
- 2
 - a $a = 130^\circ$, angles in a full turn total 360° .
 $b = 65^\circ$, the angle at the centre of a circle is twice the angle at the circumference.
 $c = 115^\circ$, opposite angles in a cyclic quadrilateral total 180° .
 - b $d = 36^\circ$, isosceles triangle.
 $e = 108^\circ$, angles in a triangle total 180° .
 $f = 54^\circ$, angle in a semicircle is 90° .
 - c $g = 127^\circ$, angles at a full turn total 360° , the angle at the centre of a circle is twice the angle at the circumference.
 - d $h = 36^\circ$, the angle at the centre of a circle is twice the angle at the circumference.
- 3
 - a $a = 25^\circ$, angles in the same segment are equal.
 $b = 45^\circ$, angles in the same segment are equal.
 - b $c = 44^\circ$, angles in the same segment are equal.
 $d = 46^\circ$, the angle in a semicircle is 90° and the angles in a triangle total 180° .
 - c $e = 48^\circ$, the angle at the centre of a circle is twice the angle at the circumference.
 $f = 48^\circ$, angles in the same segment are equal.
 - d $g = 100^\circ$, angles at a full turn total 360° , the angle at the centre of a circle is twice the angle at the circumference.
 $h = 100^\circ$, angles in the same segment are equal.
- 4
 - a $a = 75^\circ$, opposite angles in a cyclic quadrilateral total 180° .
 $b = 105^\circ$, angles on a straight line total 180° .
 $c = 94^\circ$, opposite angles in a cyclic quadrilateral total 180° .
 - b $d = 92^\circ$, opposite angles in a cyclic quadrilateral total 180° .
 $e = 88^\circ$, angles on a straight line total 180° .
 $f = 92^\circ$, angles in the same segment are equal.
 - c $h = 80^\circ$, alternate segment theorem.
 - d $g = 35^\circ$, alternate segment theorem and the angle in a semicircle is 90° .

5 Angle $BAT = x$.

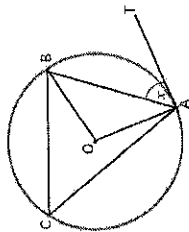
Angle $OAB = 90^\circ - x$ because the angle between the tangent and the radius is 90° .

$OA = OB$ because radii are equal.

Angle $OAB =$ angle OBA because the base of isosceles triangles are equal.

Angle $AOB = 180^\circ - (90^\circ - x) - (90^\circ - x) = 2x$ because angles in a triangle total 180° .

Angle $ACB = 2x \div 2 = x$ because the angle at the centre is twice the angle at the circumference.



Trigonometry in right-angled triangles

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
 - the side opposite the right angle is called the hypotenuse
 - the side opposite the angle θ is called the opposite
 - the side next to the angle θ is called the adjacent.
- In a right-angled triangle:
 - the ratio of the opposite side to the hypotenuse is the sine of angle θ , $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
 - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ , $\cos \theta = \frac{\text{adj}}{\text{hyp}}$
 - the ratio of the opposite side to the adjacent side is the tangent of angle θ , $\tan \theta = \frac{\text{opp}}{\text{adj}}$
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.
- The sine, cosine and tangent of some angles may be written exactly.

	0°	30°	45°	60°	90°
$\sin$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

Examples

Example 1

Calculate the length of side x .
Give your answer correct to 3 significant figures.



- Always start by labelling the sides.
- You are given the adjacent and the hypotenuse so use the cosine ratio.
- Substitute the sides and angle into the cosine ratio.
- Rearrange to make x the subject.
- Use your calculator to work out $6 \div \cos 25^\circ$.
- Round your answer to 3 significant figures and write the units in your answer.

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\cos 25^\circ = \frac{6}{x}$$

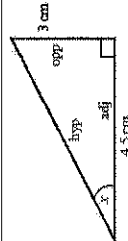
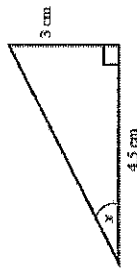
$$x = \frac{6}{\cos 25^\circ}$$

$$x = 6.620\ 267\ 5...$$

$$x = 6.62\ \text{cm}$$

Example 2

Calculate the size of angle x .
Give your answer correct to 3 significant figures.



- Always start by labelling the sides.
- You are given the opposite and the adjacent so use the tangent ratio.
- Substitute the sides and angle into the tangent ratio.
- Use $\tan^{-1}$ to find the angle.
- Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
- Round your answer to 3 significant figures and write the units in your answer.

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

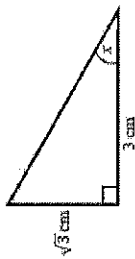
$$\tan x = \frac{3}{4.5}$$

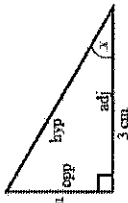
$$x = \tan^{-1}\left(\frac{3}{4.5}\right)$$

$$x = 33.690\ 067\ 5...$$

$$x = 33.7^\circ$$

Example 3 Calculate the exact size of angle x .





1 Always start by labelling the sides.

2 You are given the opposite and the adjacent so use the tangent ratio.

3 Substitute the sides and angle into the tangent ratio.

4 Use the table from the key points to find the angle.

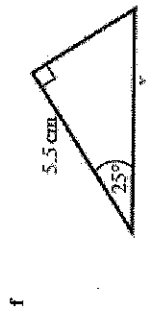
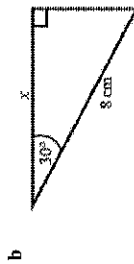
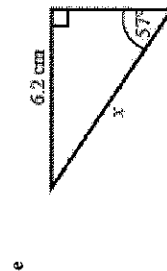
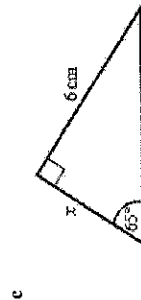
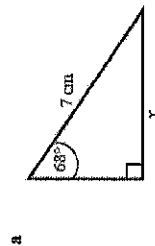
$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

$$\tan x = \frac{\sqrt{3}}{3}$$

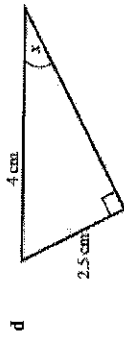
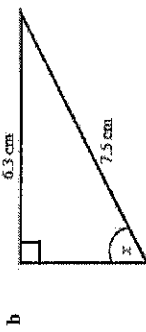
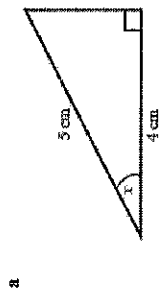
$$x = 30^\circ$$

Practice

- 1 Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

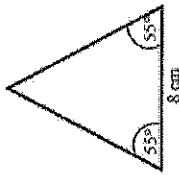


- 2 Calculate the size of angle x in each triangle. Give your answers correct to 1 decimal place.



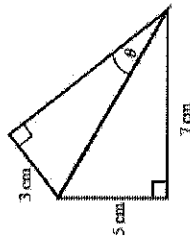
- 3 Work out the height of the isosceles triangle. Give your answer correct to 3 significant figures.

Hint:
Split the triangle into two right-angled triangles.

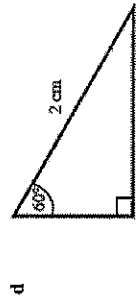
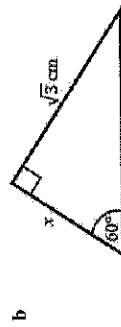
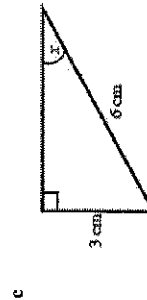
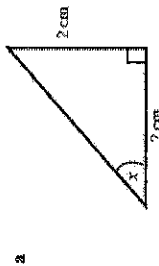


- 4 Calculate the size of angle θ . Give your answer correct to 1 decimal place.

Hint:
First work out the length of the common side to both triangles, leaving your answer in surd form.



- 5 Find the exact value of x in each triangle.



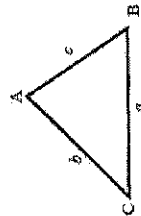
The cosine rule

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.1 The cosine rule

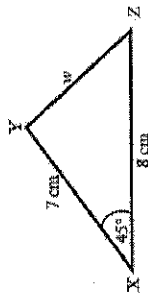
Key points

- a is the side opposite angle A .
- b is the side opposite angle B .
- c is the side opposite angle C .
- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc \cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.



Examples

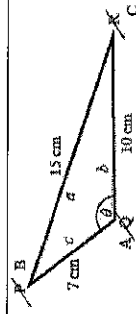
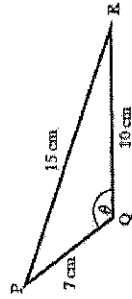
Example 4 Work out the length of side w .
Give your answer correct to 3 significant figures.



	1 Always start by labelling the angles and sides. 2 Write the cosine rule to find the side. 3 Substitute the values a, b and A into the formula. 4 Use a calculator to find w^2 and then w. 5 Round your final answer to 3 significant figures and write the units in your answer.
--	---

$a^2 = b^2 + c^2 - 2bc \cos A$
 $w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45^\circ$
 $w^2 = 33.804\ 040\ 51\dots$
 $w = \sqrt{33.804\ 040\ 51}$
 $w = 5.81\text{ cm}$

Example 5 Work out the size of angle θ .
Give your answer correct to 1 decimal place.



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos \theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$$

$$\cos \theta = \frac{-76}{140}$$

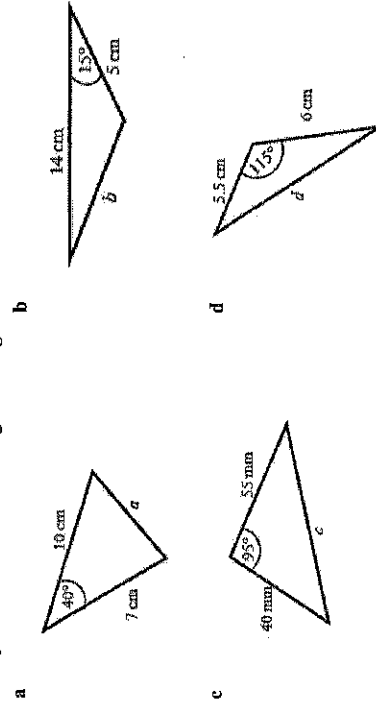
$$\theta = 122.878\ 349\dots$$

$$\theta = 122.9^\circ$$

- 1 Always start by labelling the angles and sides.
- 2 Write the cosine rule to find the angle.
- 3 Substitute the values a , b and c into the formula.
- 4 Use $\cos^{-1}$ to find the angle.
- 5 Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
- 6 Round your answer to 1 decimal place and write the units in your answer.

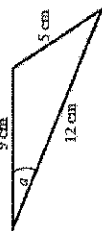
Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

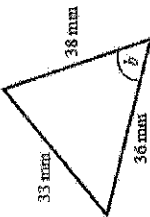


- 7 Calculate the angles labelled θ in each triangle. Give your answer correct to 1 decimal place.

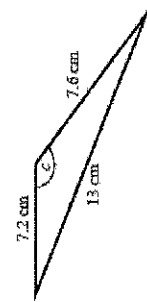
a



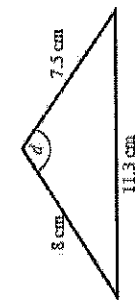
b



c

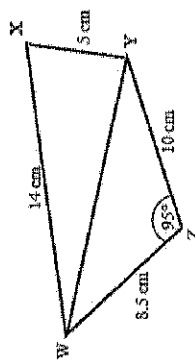


d



- 8 a Work out the length of WY. Give your answer correct to 3 significant figures.

- b Work out the size of angle WXY. Give your answer correct to 1 decimal place.



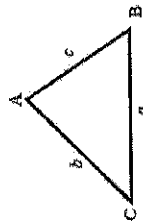
The sine rule

A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.2 The sine rule

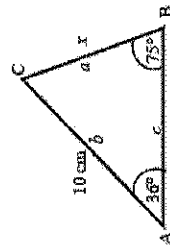
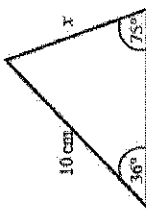
Key points

- a is the side opposite angle A .
 b is the side opposite angle B .
 c is the side opposite angle C .
- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.



Examples

- Example 6** Work out the length of side x .
Give your answer correct to 3 significant figures.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{10}{\sin 36^\circ} = \frac{x}{\sin 75^\circ}$$

$$x = \frac{10 \times \sin 75^\circ}{\sin 36^\circ}$$

$$x = 6.09 \text{ cm}$$

- 1 Always start by labelling the angles and sides.

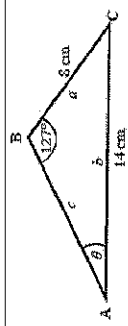
- 2 Write the sine rule to find the side.

- 3 Substitute the values a , b , A and B into the formula.

- 4 Rearrange to make x the subject.

- 5 Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ .
Give your answer correct to 1 decimal place.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{8} = \frac{\sin 127^\circ}{14}$$

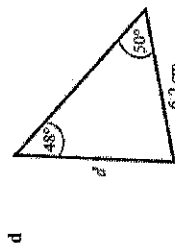
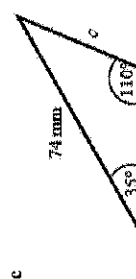
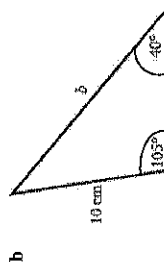
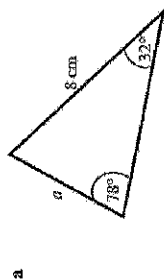
$$\sin \theta = \frac{8 \times \sin 127^\circ}{14}$$

$$\theta = 27.2^\circ$$

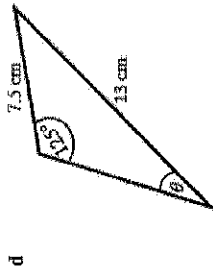
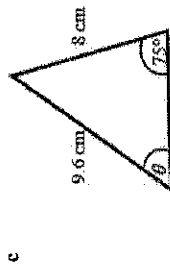
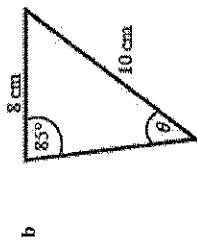
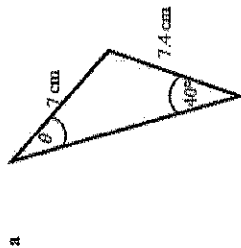
- 1 Always start by labelling the angles and sides.
- 2 Write the sine rule to find the angle.
- 3 Substitute the values a , b , A and B into the formula.
- 4 Rearrange to make $\sin \theta$ the subject.
- 5 Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

Practice

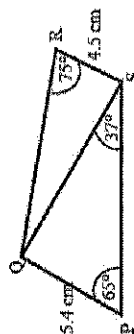
9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.



10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.



11 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.



b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

Areas of triangles

A LEVEL LINKS

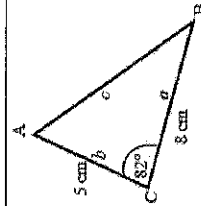
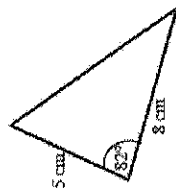
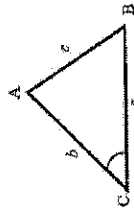
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.3 Areas of triangles

Key points

- a is the side opposite angle A .
- b is the side opposite angle B .
- c is the side opposite angle C .
- The area of the triangle is $\frac{1}{2}ab \sin C$.

Examples

Example 8 Find the area of the triangle.



$$\text{Area} = \frac{1}{2} ab \sin C$$

$$\text{Area} = \frac{1}{2} \times 8 \times 5 \times \sin 82^\circ$$

$$\text{Area} = 19.805361\dots$$

$$\text{Area} = 19.8 \text{ cm}^2$$

- 1 Always start by labelling the sides and angles of the triangle.

- 2 State the formula for the area of a triangle.

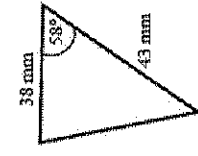
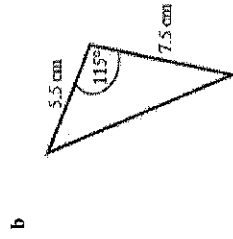
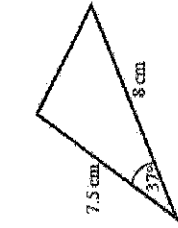
- 3 Substitute the values of a , b and C into the formula for the area of a triangle.

- 4 Use a calculator to find the area.

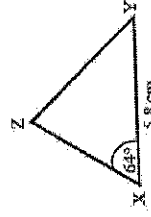
- 5 Round your answer to 3 significant figures and write the units in your answer.

Practice

- 12 Work out the area of each triangle. Give your answers correct to 3 significant figures.



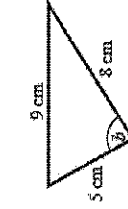
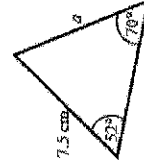
- 13 The area of triangle XYZ is 13.3 cm^2 . Work out the length of XZ .



Hint:
Rearrange the formula to make a side the subject.

Extend

- 14 Find the size of each lettered angle or side. Give your answers correct to 3 significant figures.

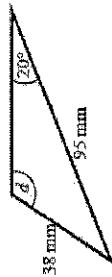


Hint:
For each one, decide whether to use the cosine or sine rule.

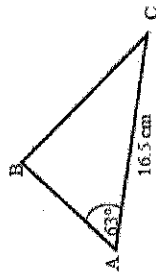
c



d



- 15 The area of triangle ABC is 86.7 cm^2 .
Work out the length of BC.
Give your answer correct to 3 significant figures.



Answers

- 1 a 6.49 cm
d 74.3 mm
b 6.93 cm
e 7.39 cm
c 2.80 cm
f 6.07 cm
- 2 a 36.9°
b 57.1°
c 47.0°
d 38.7°
- 3 5.71 cm
- 4 20.4°
- 5 a 45°
b 1 cm
c 30°
d $\sqrt{5} \text{ cm}$
- 6 a 6.46 cm
b 9.26 cm
c 70.8 mm
d 9.70 cm
- 7 a 22.2°
b 52.9°
c 122.9°
d 93.6°
- 8 a 13.7 cm
b 76.0°
- 9 a 4.33 cm
b 15.0 cm
c 45.2 mm
d 6.39 cm
- 10 a 42.8°
b 52.8°
c 53.6°
d 28.2°
- 11 a 8.13 cm
b 32.3°
- 12 a 18.1 cm^2
b 18.7 cm^2
c 693 mm^2
- 13 5.10 cm
- 14 a 6.29 cm
b 84.3°
c 5.73 cm
d 58.8°
- 15 15.3 cm

Rearranging equations

A LEVEL LINKS

Scheme of work: 6a. Definition, differentiating polynomials, second derivatives
Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$	1 Get the terms containing t on one side and everything else on the other side.
$v - u = at$	2 Divide throughout by a .
$t = \frac{v - u}{a}$	

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$	1 All the terms containing t are already on one side and everything else is on the other side.
$r = t(2 - \pi)$	2 Factorise as t is a common factor.
$t = \frac{r}{2 - \pi}$	3 Divide throughout by $2 - \pi$.

Example 3 Make t the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$	1 Remove the fractions first by multiplying throughout by 10.
$2t + 2r = 15t$	2 Get the terms containing t on one side and everything else on the other side and simplify.
$2r = 13t$	3 Divide throughout by 13.
$t = \frac{2r}{13}$	

Example 4 Make t the subject of the formula $r = \frac{3t+5}{t-1}$.

$r = \frac{3t+5}{t-1}$	1 Remove the fraction first by multiplying throughout by $t-1$.
$r(t-1) = 3t+5$	2 Expand the brackets.
$rt-r = 3t+5$	3 Get the terms containing t on one side and everything else on the other side.
$rt-3t = 5+r$	4 Factorise the LHS as t is a common factor.
$t(r-3) = 5+r$	5 Divide throughout by $r-3$.
$t = \frac{5+r}{r-3}$	

Practice

Change the subject of each formula to the letter given in the brackets.

- $C = \pi d$ [d]
- $P = 2l + 2w$ [w]
- $D = \frac{S}{T}$
- $p = \frac{q-r}{t}$ [t]
- $u = at - \frac{1}{2}t$ [t]
- $V = ax + 4x$ [x]
- $\frac{y-7x}{2} = \frac{7-2y}{3}$ [y]
- $x = \frac{2a-1}{3-a}$ [a]
- $x = \frac{b-c}{d}$
- $h = \frac{7g-9}{2+g}$ [g]
- $e(9+x) = 2e+1$ [e]
- $y = \frac{2x+3}{4-x}$ [x]
- Make r the subject of the following formulae.
 - $A = \pi r^2$
 - $V = \frac{4}{3}\pi r^3$
 - $P = \pi r + 2r$
 - $V = \frac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

- $\frac{xy}{z} = \frac{ab}{cd}$
- $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

- $\frac{p}{q}(sx+t) = x-1$
- $\frac{p}{q}(ax+2y) = \frac{3p}{q^2}(x-y)$

Answers

- 1 $d = \frac{C}{\pi}$
- 2 $w = \frac{P-2l}{2}$
- 3 $T = \frac{S}{D}$
- 4 $t = \frac{q-r}{p}$
- 5 $t = \frac{2u}{2a-1}$
- 6 $x = \frac{V}{a+4}$
- 7 $y = 2 + 3x$
- 8 $a = \frac{3x+1}{x+2}$
- 9 $d = \frac{b-c}{x}$
- 10 $g = \frac{2h+9}{7-h}$
- 11 $e = \frac{1}{x+7}$
- 12 $x = \frac{4y-3}{2+y}$
- 13 a $r = \sqrt{\frac{A}{\pi}}$
- b $r = \sqrt[3]{\frac{3V}{4\pi}}$
- c $r = \frac{P}{\pi+2}$
- d $r = \sqrt{\frac{3V}{2\pi h}}$
- 14 a $x = \frac{abz}{cdy}$
- b $x = \frac{3dz}{4\pi cpy^2}$
- 15 $\sin B = \frac{b \sin A}{a}$
- 16 $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
- 17 a $x = \frac{q+pt}{q-ps}$
- b $x = \frac{3py+2pqy}{3p-apq} = \frac{y(3+2q)}{3-aq}$

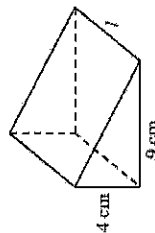
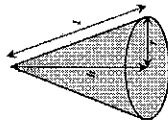
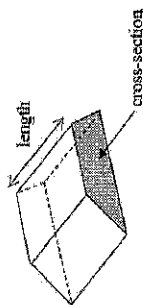
Volume and surface area of 3D shapes

A LEVEL LINKS

Scheme of work: 6b. Gradients, tangents, normals, maxima and minima

Key points

- Volume of a prism = cross-sectional area $\times$ length.
- The surface area of a 3D shape is the total area of all its faces.
- Volume of a pyramid = $\frac{1}{3} \times$ area of base $\times$ vertical height.
- Volume of a cylinder = $\pi r^2 h$
- Total surface area of a cylinder = $2\pi r^2 + 2\pi rh$
- Volume of a sphere = $\frac{4}{3} \pi r^3$
- Surface area of a sphere = $4\pi r^2$
- Volume of a cone = $\frac{1}{3} \pi r^2 h$
- Total surface area of a cone = $\pi r l + \pi r^2$



Examples

Example 1 The triangular prism has volume 504 cm^3 .
Work out its length.

$$V = \frac{1}{2} bhl$$

$$504 = \frac{1}{2} \times 9 \times 4 \times l$$

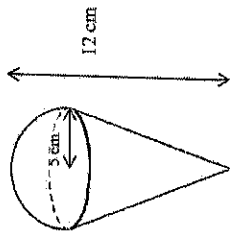
$$504 = 18 \times l$$

$$l = 504 \div 18$$

$$= 28 \text{ cm}$$

- 1 Write out the formula for the volume of a triangular prism.
- 2 Substitute known values into the formula.
- 3 Simplify
- 4 Rearrange to work out l .
- 5 Remember the units.

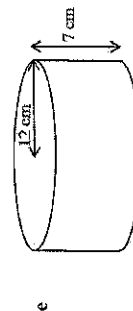
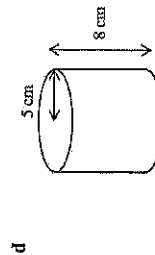
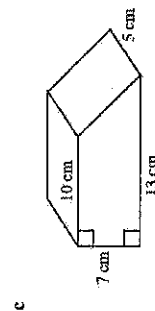
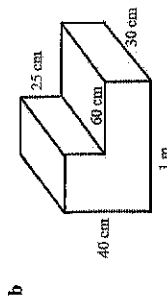
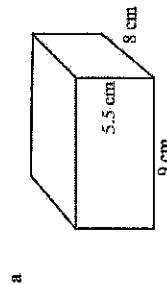
Example 2 Calculate the volume of the 3D solid.
Give your answer in terms of π .



Total volume = volume of hemisphere + Volume of cone $= \frac{1}{2} \text{ of } \frac{4}{3} \pi r^3 + \frac{1}{3} \pi r^2 h$	1 The solid is made up of a hemisphere radius 5 cm and a cone with radius 5 cm and height $12 - 5 = 7 \text{ cm}$.
Total volume = $\frac{1}{2} \times \frac{4}{3} \times \pi \times 5^3$ + $\frac{1}{3} \times \pi \times 5^2 \times 7$ $= \frac{425}{3} \pi \text{ cm}^3$	2 Substitute the measurements into the formula for the total volume.
	3 Remember the units.

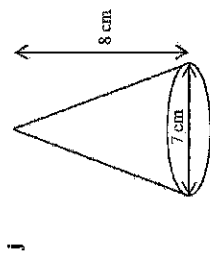
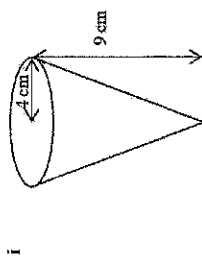
Practice

- 1 Work out the volume of each solid.
Leave your answers in terms of π where appropriate.



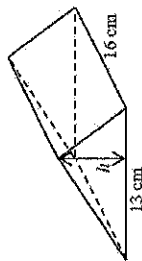
f a sphere with radius 7 cm

- g a sphere with diameter 9 cm



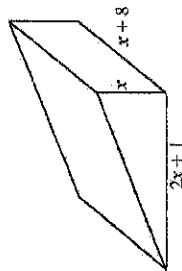
- 2 A cuboid has width 9.5 cm, height 8 cm and volume 1292 cm^3 .
Work out its length.

- 3 The triangular prism has volume 1768 cm^3 .
Work out its height.

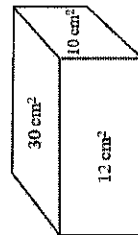


Extend

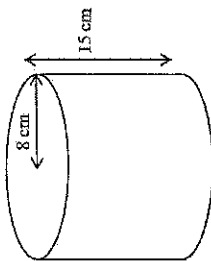
- 4 The diagram shows a solid triangular prism.
All the measurements are in centimetres.
The volume of the prism is $V \text{ cm}^3$.
Find a formula for V in terms of x .
Give your answer in simplified form.



- 5 The diagram shows the area of each of three faces of a cuboid.
The length of each edge of the cuboid is a whole number of centimetres.
Work out the volume of the cuboid.

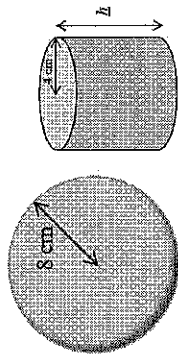


- 6 The diagram shows a large catering size tin of beans in the shape of a cylinder.

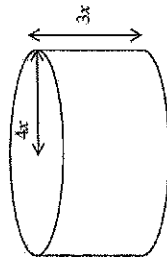


The tin has a radius of 8 cm and a height of 15 cm.
A company wants to make a new size of tin.
The new tin will have a radius of 6.7 cm.
It will have the same volume as the large tin.
Calculate the height of the new tin.
Give your answer correct to one decimal place.

- 7 The diagram shows a sphere and a solid cylinder.
The sphere has radius 8 cm.
The solid cylinder has a base radius of 4 cm and a height of h cm.
The total surface area of the cylinder is half the total surface area of the sphere.
Work out the ratio of the volume of the sphere to the volume of the cylinder.
Give your answer in its simplest form.



- 8 The diagram shows a solid metal cylinder.
The cylinder has base radius $4x$ and height $3x$.
The cylinder is melted down and made into a sphere of radius r .
Find an expression for r in terms of x .



Answers

- | | | | | |
|----------|----------|-----------------------------|----------|--------------------------------------|
| 1 | a | $V = 396 \text{ cm}^3$ | b | $V = 75\,000 \text{ cm}^3$ |
| | c | $V = 402.5 \text{ cm}^3$ | d | $V = 200\pi \text{ cm}^3$ |
| | e | $V = 1008\pi \text{ cm}^3$ | f | $V = \frac{1372}{3}\pi \text{ cm}^3$ |
| | g | $V = 121.5\pi \text{ cm}^3$ | h | $V = 18\pi \text{ cm}^3$ |
| | i | $V = 48\pi \text{ cm}^3$ | j | $V = \frac{98}{3}\pi \text{ cm}^3$ |
-
- | | |
|----------|----------------------------------|
| 2 | 17 cm |
| 3 | 17 cm |
| 4 | $V = x^3 + \frac{17}{2}x^2 + 4x$ |
| 5 | 60 cm ³ |
| 6 | 21.4 cm |
| 7 | 32 : 9 |
| 8 | $r = \sqrt[3]{36x}$ |

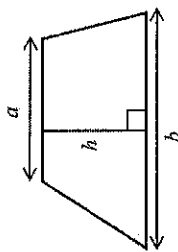
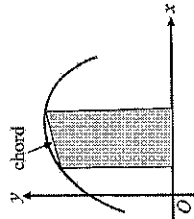
Area under a graph

A LEVEL LINKS

Scheme of work: 7b. Definite integrals and areas under curves

Key points

- To estimate the area under a curve, draw a chord between the two points you are finding the area between and straight lines down to the horizontal axis to create a trapezium. The area of the trapezium is an approximation for the area under a curve.

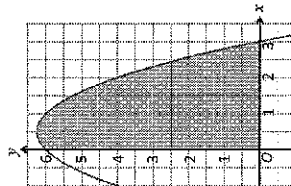


- The area of a trapezium = $\frac{1}{2}h(a+b)$

Examples

Example 1

Estimate the area of the region between the curve $y = (3-x)(2+x)$ and the x-axis from $x = 0$ to $x = 3$.
Use three strips of width 1 unit.



x	0	1	2	3
y = (3-x)(2+x)	6	6	4	0

Trapezium 1:

$$a_1 = 6 - 0 = 6, \quad b_1 = 6 - 0 = 6$$

Trapezium 2:

$$a_2 = 6 - 0 = 6, \quad b_2 = 4 - 0 = 4$$

Trapezium 3:

$$a_3 = 4 - 0 = 4, \quad b_3 = 0 - 0 = 0$$

- Use a table to record the value of y on the curve for each value of x .

- Work out the dimensions of each trapezium. The distances between the y -values on the curve and the x -axis give the values for a .

(continued on next page)

$$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 1(6 + 6) = 6$$

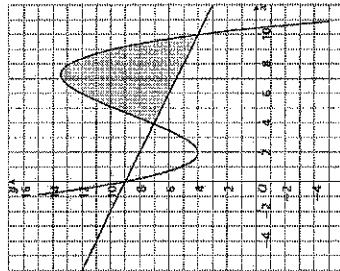
$$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 1(6 + 4) = 5$$

$$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 1(4 + 0) = 2$$

$$\text{Area} = 6 + 5 + 2 = 13 \text{ units}^2$$

- Work out the area of each trapezium. $h = 1$ since the width of each trapezium is 1 unit.

- Work out the total area. Remember to give units with your answer.



Estimate the shaded area.

Use three strips of width 2 units.

Example 2

- Use a table to record y on the curve for each value of x .

- Use a table to record y on the straight line for each value of x .

- Work out the dimensions of each trapezium. The distances between the y -values on the curve and the y -values on the straight line give the values for a .

- Work out the area of each trapezium. $h = 2$ since the width of each trapezium is 2 units.

- Work out the total area. Remember to give units with your answer.

x	4	6	8	10
y	7	12	13	4

x	4	6	8	10
y	7	6	5	4

Trapezium 1:

$$a_1 = 7 - 7 = 0, \quad b_1 = 12 - 6 = 6$$

Trapezium 2:

$$a_2 = 12 - 6 = 6, \quad b_2 = 13 - 5 = 8$$

Trapezium 3:

$$a_3 = 13 - 5 = 8, \quad b_3 = 4 - 4 = 0$$

$$\frac{1}{2}h(a_1 + b_1) = \frac{1}{2} \times 2(0 + 6) = 6$$

$$\frac{1}{2}h(a_2 + b_2) = \frac{1}{2} \times 2(6 + 8) = 14$$

$$\frac{1}{2}h(a_3 + b_3) = \frac{1}{2} \times 2(8 + 0) = 8$$

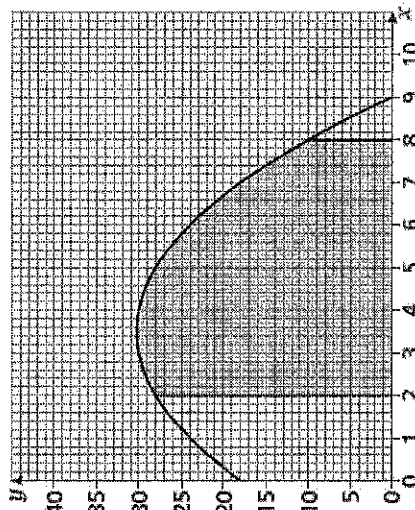
$$\text{Area} = 6 + 14 + 8 = 28 \text{ units}^2$$

Practice

- 1 Estimate the area of the region between the curve $y = (5 - x)(x + 2)$ and the x -axis from $x = 1$ to $x = 5$.
Use four strips of width 1 unit.

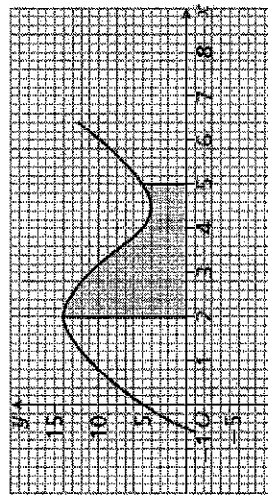
Hint:

For a full answer, remember to include 'units²'.



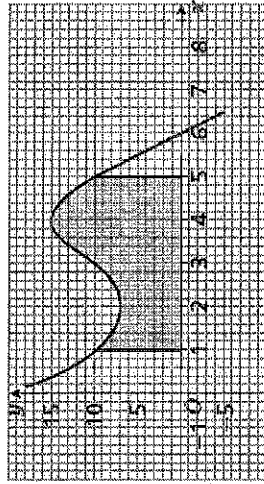
- 2 Estimate the shaded area shown on the axes.
Use six strips of width 1 unit.

- 3 Estimate the area of the region between the curve $y = x^2 - 8x + 18$ and the x -axis from $x = 2$ to $x = 6$.
Use four strips of width 1 unit.



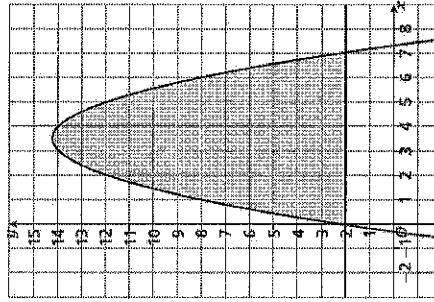
- 4 Estimate the shaded area.
Use six strips of width $\frac{1}{2}$ unit.

- 5 Estimate the area of the region between the curve $y = -x^2 - 4x + 5$ and the x -axis from $x = -5$ to $x = 1$.
Use six strips of width 1 unit.



- 6 Estimate the shaded area.
Use four strips of equal width.

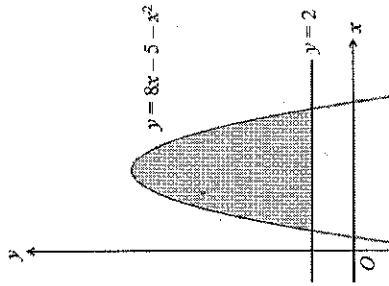
- 7 Estimate the area of the region between the curve $y = -x^2 + 2x + 15$ and the x -axis from $x = 2$ to $x = 5$.
Use six strips of equal width.



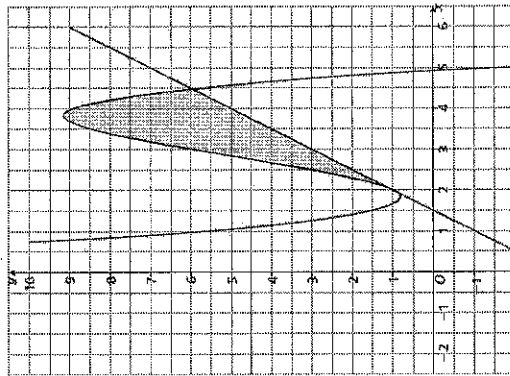
- 8 Estimate the shaded area.
Use seven strips of equal width.

Extend

- 9 The curve $y = 8x - 5 - x^2$ and the line $y = 2$ are shown in the sketch. Estimate the shaded area using six strips of equal width.



- 10 Estimate the shaded area using five strips of equal width.



Answers

- 1 34 units²
- 2 149 units²
- 3 14 units²
- 4 $25\frac{1}{4}$ units²
- 5 35 units²
- 6 42 units²
- 7 $26\frac{7}{8}$ units²
- 8 56 units²
- 9 35 units²
- 10 $6\frac{1}{4}$ units²

SECTION 2

BASELINE ASSESSMENT

Year 12 AS/A level Maths Baseline Assessment

Instructions

- The time for the test is 1 hour. PLEASE DO KEEP TO 1 HOUR
- Answer **all** questions.

Information

- The total mark for this paper is 48.
- The marks for each question are shown in brackets
-use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Please HAND IN YOUR COMPLETED TEST IN SEPTEMBER.

Remember this should be your own work, you will not be graded on this assessment, it is purely to see your understanding up to this point before we start the A Level course.

1 Simplify these expressions.

a $\frac{x^3 \times x^4}{x^2}$ (1 mark)

b $(2x^3)^4$ (1 mark)

c $\frac{9x^{\frac{1}{2}}}{(27x^{-2})^{\frac{2}{3}}}$ (3 marks)

2 Solve $2x^2 \times 4x^4 = 512$ (2 marks)

3 Find the value of x .

$x^{-\frac{4}{3}} = \frac{1}{256}$ (2 marks)

4 **a** Write $\sqrt{240}$ in the form $a\sqrt{15}$, where a is an integer. (1 mark)

b Expand and simplify $(2 - \sqrt{3})(5 + 2\sqrt{3})$. (2 marks)

c Simplify $\frac{2 + \sqrt{5}}{3 - \sqrt{5}}$ giving your answer in the form $a + b\sqrt{c}$, where a , b and c are rational numbers. (3 marks)

5 The area of a triangle is given as $(7 + 3\sqrt{3}) \text{ cm}^2$.

The base of the triangle is $(5 - \sqrt{3}) \text{ cm}$, and the perpendicular height is $(p + q\sqrt{3}) \text{ cm}$.

Find the values of p and q . (4 marks)

6 Expand and simplify these expressions.

a $3(x - 2y)$ (1 mark)

b $(2x - 3)(3x + 5)$ (2 marks)

c $(x - 2)^2(x + 5)$ (3 marks)

7 Fully factorise these expressions.

a $2xy - 4x$ (1 mark)

b $x^2 + 2x - 3$ (1 mark)

8 Solve these equations.

a $3x - 7 = 17$ (1 mark)

b $x^2 - 6x + 5 = 0$ (2 marks)

c $2x^2 - 5x + 1 = 0$ (2 marks)

9 Solve these pairs of simultaneous equations.

a $2x + y = 7$ (3 marks)

$3x - y = 8$

b $y = 3x - 1$ (3 marks)

$3y = 6x + 1$

c $2x - y = 9$ (4 marks)

$x^2 + y^2 = 17$

10 Solve these inequalities.

a $7x - 6 \leq 8$ (1 mark)

b $3x + 2 \geq 7x - 4$ (2 marks)

c $x^2 + 12x - 28 > 0$ (2 marks)

11 The function f is defined as $f(x) = 5x + 2$

Find the value of $f(-4)$. (1 mark)

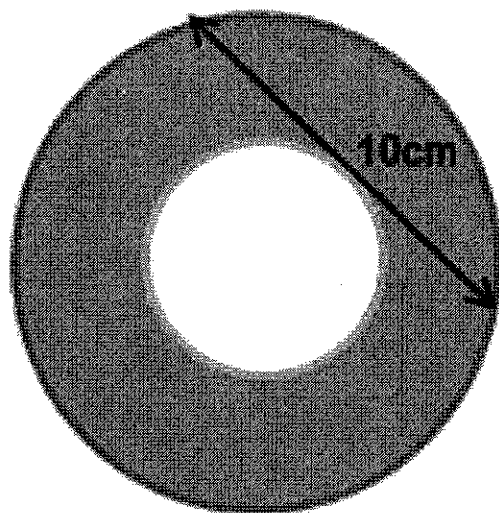
This is the end of the test.

SECTION 3

PROBLEM SOLVING

Circle areas

Can you work out the shaded area in the diagram (the line shown just touches the smaller circle)?



Find the value of

$$\frac{99}{100} \times \frac{80}{81} \times \frac{63}{64} \times \frac{48}{49} \times \frac{35}{36} \times \frac{24}{25} \times \frac{15}{16} \times \frac{8}{9} \times \frac{3}{4}.$$

Write your answer in the form $\frac{a}{b}$, where a and b are positive integers with no common factors other than 1.

A point E lies outside the rectangle $ABCD$ such that CBE is an equilateral triangle. The area of the pentagon $ABECD$ is five times the area of the triangle CBE .

What is the ratio of the lengths $AB : AD$?

Write your answer in the form $a : 1$.

A sequence is defined as follows:

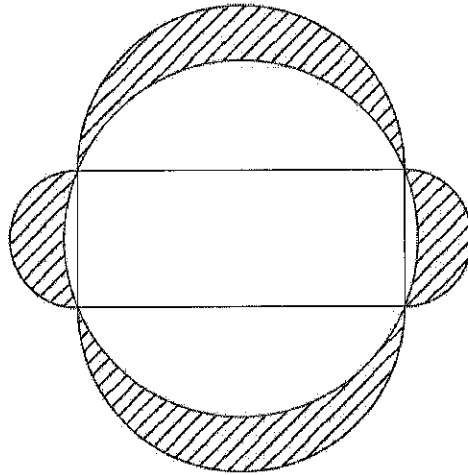
$$u_1 = 123.$$

For $n \geq 1$, define u_{n+1} = the sum of the squares of the digits of u_n .

For example, $u_2 = 1^2 + 2^2 + 3^2 = 14$, $u_3 = 1^2 + 4^2 = 17$.

What is the value of u_{100} ?

Four semicircles are drawn on the sides of a rectangle with width 10 cm and length 24 cm. A circle is drawn that passes through the four vertices of the rectangle.



What is the value, in cm^2 , of the shaded area?

Alfred, Brenda, Colin, David and Erica have to sit on a row of five chairs. Alfred does not want to sit next to Brenda. David does not want to sit next to Erica.

In how many ways can these five people arrange themselves and ensure the above conditions are met?

- (a) Which positive integer in the range from 1 to 250 has more different prime divisors than any other integer in this range? [3 marks]

- (b) When $n = 5$ the product $n(n + 1)(n + 2)$ can be written as the product of four distinct primes. Indeed, when $n = 5$

$$n(n + 1)(n + 2) = 5 \times 6 \times 7 = 2 \times 3 \times 5 \times 7.$$

What is the least positive integer n such that $n(n + 1)(n + 2)$ can be written as a product of *five* distinct primes? [3 marks]

Find the value of

$$\left(\left(2^{\frac{3}{4}} + 1 \right)^2 + \left(2^{\frac{3}{4}} - 1 \right)^2 \right) \left(\left(2^{\frac{3}{4}} + 1 \right)^2 + \left(2^{\frac{3}{4}} - 1 \right)^2 - 2^2 \right).$$

The points $A(1, 2)$ and $B(-2, 1)$ are two vertices of a rectangle $ABCD$. The diagonal CA produced passes through the point $(2, 9)$. Calculate the coordinates of the vertices C and D .

The inequalities $x^2 + 3x + 2 > 0$ and $x^2 + x < 2$ are met by all x in the region:

- (a) $x < -2$;
- (b) $-1 < x < 1$;
- (c) $x > -1$;
- (d) $x > -2$.

